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AN INVESTIGATION OF  
DYNAMIC STRESSES IN A LANDING GEAR  
AT  
A PRE-DETERMINED STRUT ANGLE

A Thesis  
Submitted to the Graduate Faculty  
of the  
University of Minnesota

by  
Burr V. <sup>W</sup>Turner

In partial fulfillment of the requirements for  
the Degree of  
Master of Science in Aeronautical Engineering  
August, 1949

THE UNIVERSITY OF  
CHICAGO PRESS

AT

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A Thesis

Submitted to the Graduate Faculty

of the

University of Chicago

by

JOHN F. DODD

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF

MASTERS OF ARTS IN THE UNIVERSITY OF CHICAGO

CHICAGO, ILL.

## PREFACE

During the history of aviation the stresses and strains which occur in landing gears during the initial part of landing an airplane have not been thoroughly investigated. In this thesis a study was made of the stresses and deflections which occurred upon landing. The response and forces resulting from these dynamic loads will be of primary concern.

The testing apparatus is located in building No. 717, Rosemount Research Center, Rosemount, Minnesota.

The reference material used in developing this theory, consisting of periodicals and engineering texts, were obtained from the aeronautical engineering office and the engineering library of the University of Minnesota.

The author is greatly indebted to Professor J. A. Wise for his guidance and valuable assistance in the preparation of this paper. Thanks is also due H. Wood for his liberal collaboration in the construction, design and testing activities.

B. V. T.

Minneapolis--August, 1949.

During the history of visiting the museum  
and various other events in London were during the initial  
part of London as a whole have not been thoroughly  
investigated. In this regard a study was made of the  
museums and collections which occurred from London.  
The museum was found to be a museum from these aspects  
London will be of great interest.

The history museum is located in London  
No. 117, Somerset House, London, England.  
The museum contains a large collection of  
history, including of various objects and documents, etc.  
were obtained from the historical research office  
and the historical library of the University of London.  
etc.

The museum is now being visited by visitors  
J. J. Allen for his research and various materials in  
the preparation of his book. There is also a  
need for the library collection in the collection,  
which has been visited.

J. J. Allen



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## SUMMARY

This is a preliminary study of the dynamic conditions of a landing gear and covers a range from light to above average landings. The weight assumed is approximately two-fifths of the normal static load on a landing gear. In actual landings an airplane wing still has lift during its initial phase, therefore, these assumptions are reasonable.

The experimental work covers the dropping range from two to five feet per second with varying tire pressures from twenty-four to forty pounds per square inch. These two parameters are the only conditions varied in this report.

A theoretical sample problem is worked for these conditions: A tire is assumed with 30 pounds per square inch pressure carrying a total weight of one thousand and sixty pounds and having a dropping velocity of four feet per second.

A comparison of this problem is made with the experimental values. The results indicate that the theoretical force is 22 per cent greater than the recorded data. The period resulting from setting up the equations of motion show that it is two and one half per cent less than that measured.

There is a stationary band of the dynamic cord  
above of a landing gear and above a large free  
to those average landing. The weight against it  
approximately two-thirds of the normal weight  
a landing gear. In some landing on a large  
still see the landing gear, however,  
these conditions are possible.

The experimental work covers the frequency range from 20 to 1000 Mc and includes the following features:

and sixty pounds and having a dropping velocity of four  
feet per second.

A comparison of hole problems is made with the experimental values. The results indicate that the maximum force is 25 per cent greater than the theoretical value. The results resulting from testing on the specimens of nylon show that it is two and one half per cent less than that measured.

A moving picture was made of the dropping operations and is available to show the action in slow motion. It is filed in Visual Education, Westbrook Hall, University of Minnesota under Aeronautical Engineering Films.

A moving picture was made of the factory during  
 bloom and is available in some of the stores in the  
 It is filed in Special Division, Washington Hall, Library  
 City of Washington under Agricultural Experimentation  
 Division is located in the building at 1000 14th Street  
 in the second floor. The building is a two-story  
 approximately 100 feet long and 40 feet wide. It  
 is located on the corner of 14th Street and  
 between Pennsylvania Avenue and the  
 Washington Monument. The building is a  
 modern structure with a flat roof and  
 large windows. It is the headquarters  
 of the Agricultural Experiment Station  
 and the Bureau of Plant Industry.  
 The building is a two-story structure  
 with a flat roof and large windows.  
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 and the Bureau of Plant Industry.  
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 and the Bureau of Plant Industry.



## INTRODUCTION

This thesis is the initial study in a projected series of reports concerning the stresses, strains, deflections and general information of a landing gear. This is part of a plan created by the Aeronautical Engineering Department, University of Minnesota. Only a limited phase of the subject will be covered in this paper since the scope of the field is far reaching. The reason for this limitation is that excessive time was required in the original construction of the testing apparatus.

First, a method had to be devised to simulate controlled landings similar to those encountered in an aircraft. This set-up was required to be in a laboratory so that accurate readings could be observed and recorded.

A Navy SNJ landing gear was used for testing purposes. The range of tire pressure was from 24 to 40 pounds per square inch, while the sinking speeds were varied from 2 to 5 feet per second. These ranges were chosen as being close to conventional landing conditions.

The present data gives sufficient information to verify the theory, but more recordings would have given a clearer picture.

This is the initial study in a projected series of reports concerning the effects of the various types of radiation on the health of man. The results of this study are being reported in the form of a preliminary report to the National Academy of Sciences, Washington, D. C., and will be published in the near future.

When I received the letter from Mr. [redacted] dated 10/10/68, I was surprised to find that he had been contacted by a person who claimed to be a member of the [redacted] and that this person had offered him \$10,000 to help in the [redacted].

A very old building that has been the subject of  
poor. The name of the property was from 18 to 19  
pounds per square inch, with the electric resistance  
ratio from 1 to 2 feet per second. These ratios were  
chosen as being close to conventional building conditions.  
The present data give sufficient information to  
verify the theory, but more recordings would have given  
a closer check.

## APPARATUS AND INSTRUMENTS

The problem required a setup which would simulate the actual landing of an aircraft. A large flywheel was desired which would withstand heavy weights and could be revolved at speeds equivalent to those of landings. When a flywheel ten feet in diameter was found, an old reduction gear and test engine were used for power. This combination took care of turning such a large disc. With this material on hand the drawings of the pit and assembly were made.

The final assembly is shown in Fig. No. 1.



Fig. No. 1



The project involved a study which would eliminate

the general feeling of an inferiority. A large number

are desired which would eliminate many subjects and could

be repeated at some point in time at Jackson.

From a physical point of view in Jackson was found, as well

as the fact that the subjects were not the same.

This condition was one of having been a large

class. With this material in hand the knowledge of the

fit and assembly were made.

The final assembly is shown in Fig. 1.



The driving mechanism is illustrated in Fig. No. 2.



Fig. No. 2

After the apparatus was constructed type C-1, SR-4, strain gauges, made by Baldwin Southwark Division of the Baldwin Locomotive Works, were placed on the lower brace of the landing gear in a to and aft position opposite each other so the drag forces could be measured. Also, gauges were placed perpendicularly to those formerly mentioned in order to measure the axial forces. To determine the relative displacements between the oleo and the strut itself a potentiometer with a scissors lever arm was installed. A cantilever with damper was placed in an appropriate position, and strain gauges were properly located to measure the movement and deflec-



tion of the wheel. These gauges can be seen in Fig. No. 3.



Fig. No. 3

The leads from the strain gauges and the potentiometer were brought into a Brush Recorder which recorded the different forces and amplitudes. The weight of the gear was determined by using platform scales and a lever system - Fig. No. 4.

stop at the hotel. There is no room in the

at the

the hotel

The hotel is a small one and the hotel is  
not very large. It is a small one and the hotel is

The hotel is a small one and the hotel is

The hotel is a small one and the hotel is

A large hotel - the hotel is

The hotel is a small one and the hotel is

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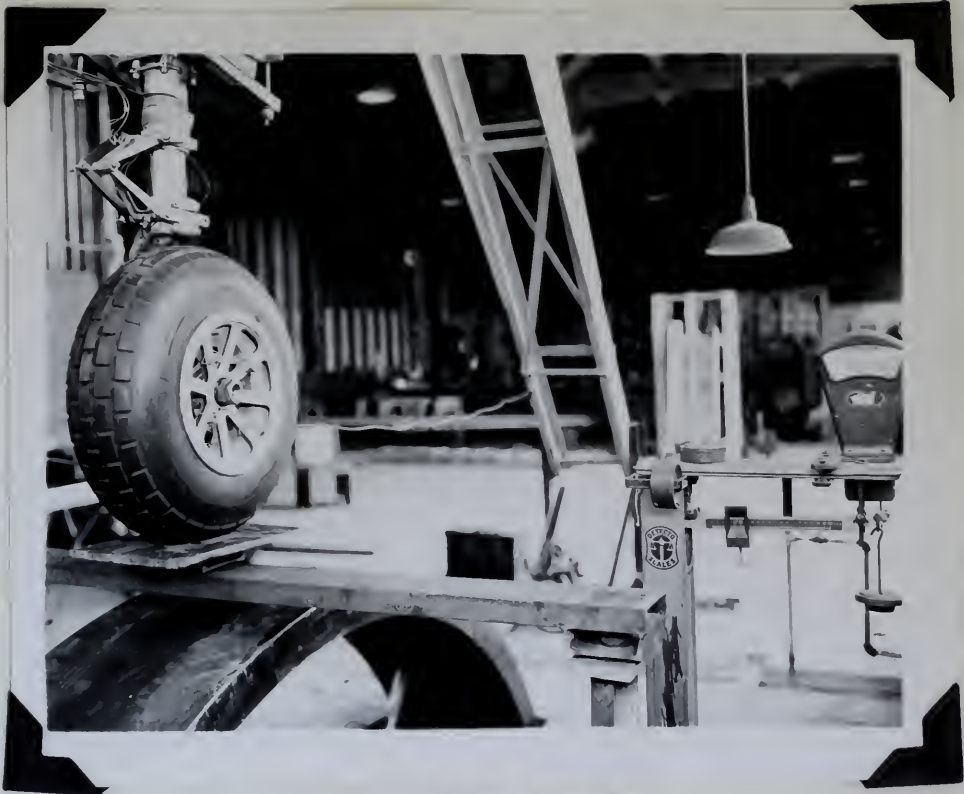


Fig. No. 4.

4 5 6

## TESTING PROCEDURE

The method used in calibrating these three pairs of strain gauges and the one potentiometer are as follows:

(A) Axial Force. The two opposite gauges on the fork running parallel to the axis of the strut measure this force. The strain analyser was balanced and no load conditions were recorded. In this manner a relationship was established between the deflection of the oscilloscope and the vertical load applied. A full static load was applied and again readings were recorded. In other words, so many millimeters of deflection on the recorder indicated a known amount of force. Points between full load and no load were checked and found in agreement.

(B) Drag Force. These forces were obtained in a manner very similar to those mentioned above. Attention should be called to the procedure in which a horizontal pressure was applied to the strut. A light cable was stretched through a pulley where a known weight could be suspended. The cable can be seen in Fig. Nos. 1, 3, and 4. Here, with known loads, the recorder was calibrated.

(C) Cantilever Deflections. The displacement of the cantilever was measured by adjusting the landing gear a known distance above the flywheel; then by

# TESTING PROCEDURE

The subject was in sitting position with feet  
of strain gauges and the two potentiometers are as  
follows:

(A) Left Foot. The two potentiometers on the  
foot resting parallel to the sole of the foot measure  
this force. The strain gauges was placed on the  
foot conditions were measured. In this manner a rela-  
tionship was established between the deflection of the  
potentiometer and the vertical force applied. A full  
static load was applied and again readings were re-  
corded. In other words, no more relationship to deflec-  
tion of the potentiometer indicated a known amount of force.  
Potentiometer (B) foot and an load were measured and  
found to be constant.

(B) Right Foot. These potentiometers were placed in a  
manner very similar to those mentioned above. Attention  
should be called to the procedure in which a horizontal  
pressure was applied to the foot. A right angle was  
applied through a pulley where a known weight could be  
applied. The cable was then in the foot. It was  
found, with known loads, the potentiometer was calibrated.

(C) Horizontal Deflection. The deflection of  
the potentiometer was measured by adjusting the loading  
from a known distance where the potentiometer was at



lowering the gear a known deflection was incurred. This deflection was recorded on the Brush Recorder. A relation between landing wheel movement and the recording instrument was established through this action.

(D) Potentiometer. The potentiometer was the only article in the instrumentation which was not linear in recording characteristics. Here the oleo was deflected a prescribed amount and recordings were made. This was necessary because the potentiometer was operated by a scissors arrangement, and even under these conditions the values approached a straight line.

Testing procedure was begun when all the instruments were tested and calibrated. In order to operate the mechanism the Essex (test engine) was started and allowed to attain a fair rate of speed before engaging the reduction gear. Due to the weight of the large flywheel it was necessary to turn it over manually before tying in the drive system. Even under these conditions there was a great deal of slipping in the belting arrangement. This slipping occurred until the flywheel reached an approximate speed of twenty-five revolutions per minute. From this point the engine assumed control and was able to develop the speeds which were obtained during this study. The speeds were measured by a strobotac.

Upon reaching certain flywheel speeds, the operation shifted to where the strain gauges and controls for dropping had been placed. A quick release mechanism

leaving the gear a more definite and improved. This  
 definition was provided as the first standard. A rela-  
 tion between loading and movement and the resulting  
 treatment was established through this system.

(b) Information. The information was the only  
 basis in the instrumentation which was not linear in  
 recording characteristics. Some data was obtained  
 a provided amount and recording time factor. This was  
 necessary because the information was obtained by a  
 various treatment, and even when some conditions  
 the value remained a constant time.

Testing procedure was based upon all the information  
 was stated and adjusted. In order to control the  
 condition the data (test results) was obtained and adjusted  
 to obtain a fair view of speed before comparing the  
 condition factor. Due to the nature of the linear system  
 it was necessary to have a very accurate before type  
 in the data system. Even when some conditions were  
 was a good deal of difference in the data system.  
 This effect was caused until the system reached an  
 approximate value of twenty-five conditions per second.  
 From this point two points were noted and the data  
 to develop the theory which was obtained during this  
 study. The results were presented by a diagram.

From results of this (linear) system, the operation  
 related to some of the data system and controls the  
 resulting was then shown. A series of data was obtained

was fastened to an overhead hoist and elevated the dropping arm to a height which would give the desired sinking speed. Drops were made from heights to represent one to seven feet per second velocity at striking.

When the dropping arm was at a desired level the apparatus was ready to simulate a landing. The Brush oscilloscopes were started and the quick release mechanism was tripped. The forces and deflections which occurred during the first few seconds of each landing were recorded. This procedure was repeated for the various dropping heights and tire pressures until the data was completed.

Motion pictures were made of drops, Fig. Nos. 10 through 12, at a rate of one hundred frames per second.



one to seven feet per second velocity of air flow.  
 standing speed. From one inch from the 50 to 100  
 dropping out as a result which would give the  
 was limited to the maximum 1000 and minimum 100

[illegible][illegible]



## DISCUSSION

A strut angle of twenty-four degrees was chosen as the optimum angle of suspension. It was used as a point of departure for this thesis work and was suggested from another thesis developed simultaneously by H. Wood.<sup>1</sup> Only the forces up and down the strut are considered in developing the theory. This decision was made since the gear was in what was estimated to be the best angle.

The horizontal force problem which is neglected in this study is mentioned in the following remarks:

First: This force may be solved by equating internal energy to the external energy, but varying sectional moments of inertia pose another condition.

Second: In noting Fig. Nos. 8 to 23 inclusive, the drag and axial forces act in a uniform manner. Studying the areas under the force time curves might lead to a solution.

Third: A dynamic study of bearing friction and friction when striking contacts are made, may point to an answer of the drag force resulting from landing. Drag force is a function of friction and is of major

---

<sup>1</sup> "A Study of Dynamic Forces in Aircraft Landing Gear Struts with Relation to the Optimum Angle of Suspension", H. Wood, A Thesis for Degree of Master of Science in Aeronautical Engineering, July, 1949.

A direct study of the system of forces and motion as the system exists in nature. It was used as a basis of discussion for this study and was suggested from another point of view by H. Wood.

Only the forces on the body are considered in discussing the theory. This method was used since the rest was in fact not required to do the work. The theoretical force system which is suggested is

This study is contained in the following sections:  
 First: This force may be stated by stating the energy to the system energy, but perhaps essential elements of the system are not needed.

Second: In making the study of the system, the drag and total forces are in a uniform manner, studying the system under the force line curves and lead to a solution.

Third: A dynamic study of the system and relation with existing systems are made, and also in an aspect of the force resulting from finding. The force is a function of the system and is at a major

---

1. A study of dynamic forces in systems leading to a study of the system as the system exists in nature. H. Wood, a study of the system of forces in a dynamic system, July, 1917.

importance. A system of vectors and a knowledge of abrasion might suggest a solution.

The damping constant of the oleo was computed as follows for all the runs taken at twenty-four degrees. The axial force was known at a given time along with the spring constant of the strut. The amount by which the landing gear deflected for the above period of time was also known. These, too, were obtained from Fig. Nos. 8 through 23. Since

$$F = CX + kX$$

C can be obtained. Following this the average value of C from all the readings was computed.

The spring constants for the system were computed by applying static loads to the gear and measuring the deflections of the tire and oleo. By repeating this process a sufficient number of times a rather constant graph was obtained, Fig. No. 7.

The original proposal of this thesis was to study the variations of five parameters, but because of mechanical failure of the driving apparatus only two--dropping velocity and tire pressure--were investigated.

In the design of the apparatus a curvilinear drop is used instead of a true vertical motion. A pivot point is located in line with the top of the large fly-wheel joining a dropping arm. Therefore, from the time the tire initially touches the landing surface and until



...the ... ..

The dumping operation of the oil was completed at  
 10:00 PM. The oil was then taken to the  
 The main point was to have a clear view of the  
 which is constant of the wind. The amount of oil  
 landing gear collected for the same period of time was  
 also noted. When, not, were obtained from the  
 2. 10:00 PM. 10:00 PM.

254 ID 8 9C

There are several other points to be noted. The first is that the system is not a simple one. It is a complex system which involves a number of different components. The second point is that the system is not a simple one. It is a complex system which involves a number of different components. The third point is that the system is not a simple one. It is a complex system which involves a number of different components.



the strut is completely deflected, the landing strut changes angle from one to two degrees. The amount of change depends on the original angle setting. The suspension angle is the angle referred to when the scissors of the gear are closed.

the effect is actively followed, the learning which  
 changes with time is in two degrees. The second is  
 always depends on the initial angle of attack. The  
 comparison angle is the angle which is shown in  
 relation to the first angle.

The first angle is the angle of attack. The second  
 angle is the angle of attack. The third angle is the  
 angle of attack. The fourth angle is the angle of  
 attack. The fifth angle is the angle of attack.

The sixth angle is the angle of attack. The seventh  
 angle is the angle of attack. The eighth angle is the  
 angle of attack. The ninth angle is the angle of  
 attack. The tenth angle is the angle of attack.

The eleventh angle is the angle of attack. The  
 twelfth angle is the angle of attack. The thirteenth  
 angle is the angle of attack. The fourteenth angle is  
 the angle of attack. The fifteenth angle is the angle  
 of attack.

The sixteenth angle is the angle of attack. The  
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 eighteenth angle is the angle of attack. The  
 nineteenth angle is the angle of attack. The  
 twentieth angle is the angle of attack.

The twenty-first angle is the angle of attack. The  
 twenty-second angle is the angle of attack. The  
 twenty-third angle is the angle of attack. The  
 twenty-fourth angle is the angle of attack. The  
 twenty-fifth angle is the angle of attack.

The twenty-sixth angle is the angle of attack. The  
 twenty-seventh angle is the angle of attack. The  
 twenty-eighth angle is the angle of attack. The  
 twenty-ninth angle is the angle of attack. The  
 thirtieth angle is the angle of attack.



Oleo Deflection in inches.

Figure No. 5.

Calibration Curve for  
Potentiometer

x - Run No. 1

o - Run No. 2

Brush Recordings in millimeters

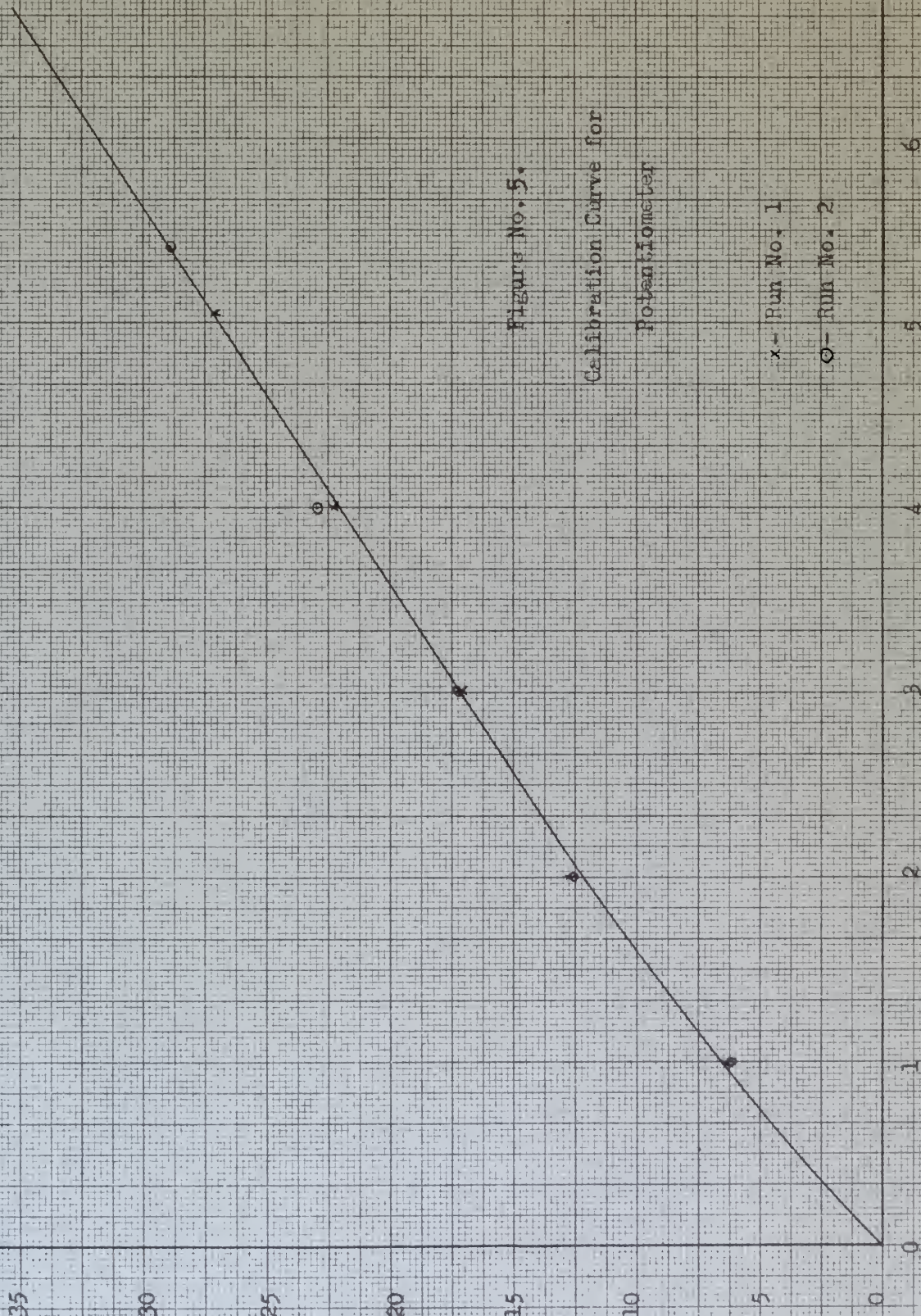
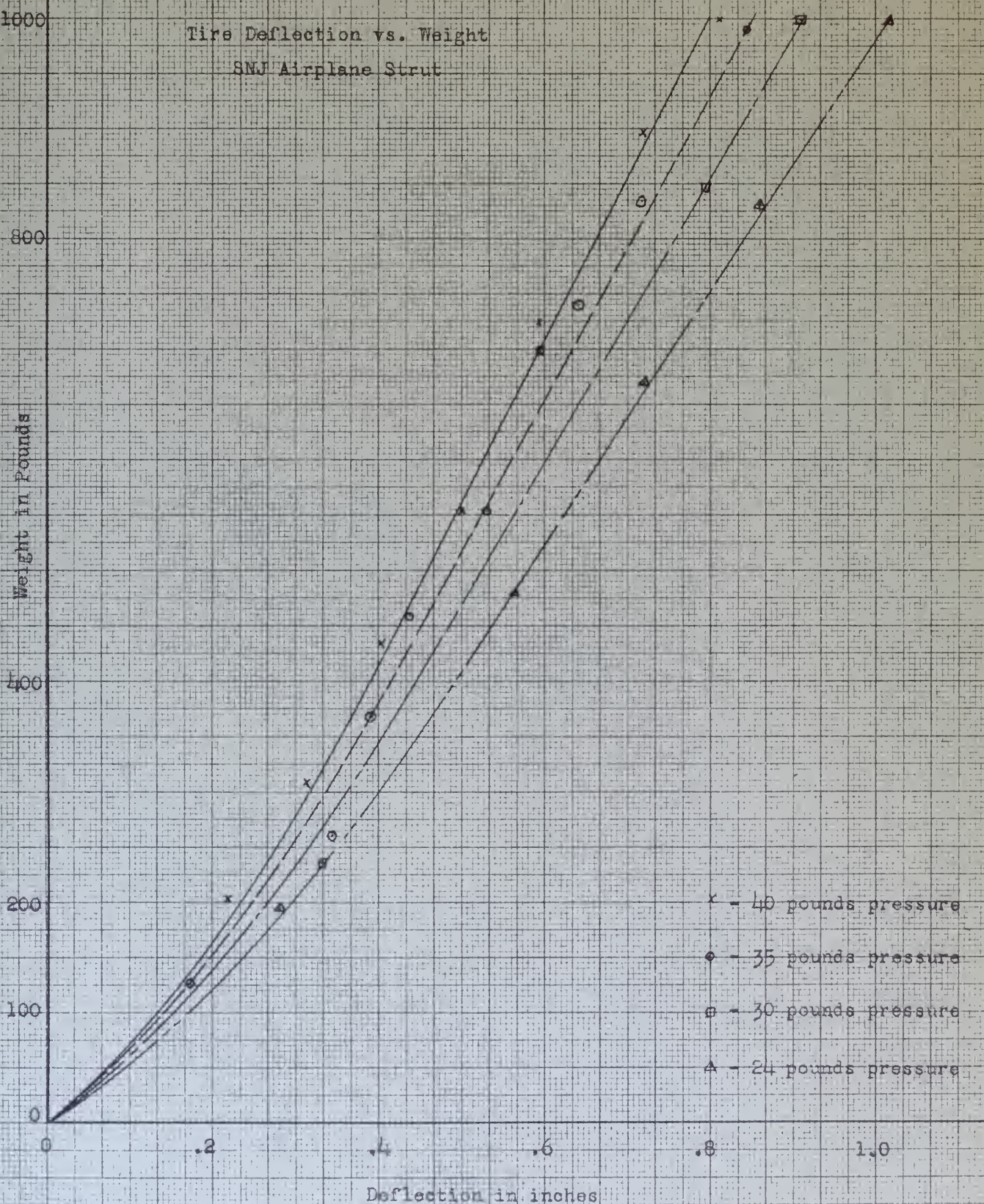






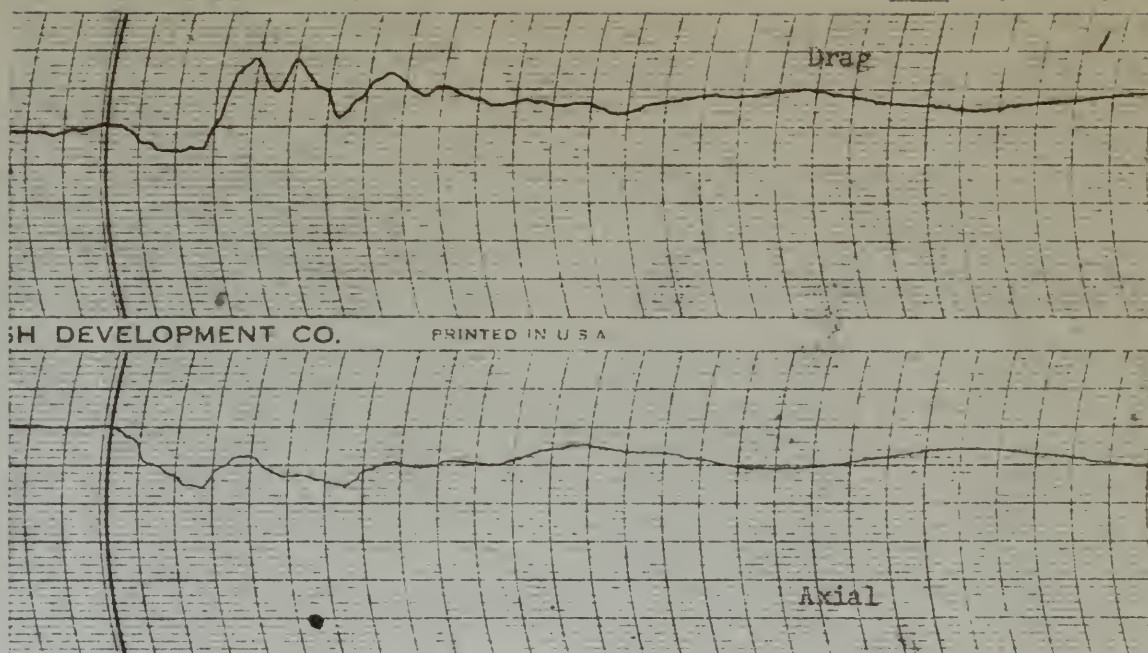
Figure No. 6

Tire Deflection vs. Weight  
SNJ Airplane Strut









Calibration:

Drag - 1 mm - 220#

Axial - 5 mm = 930#

Cantilever - 1 mm = .375"

Potentiometer - Refer to Fig. No. 5.

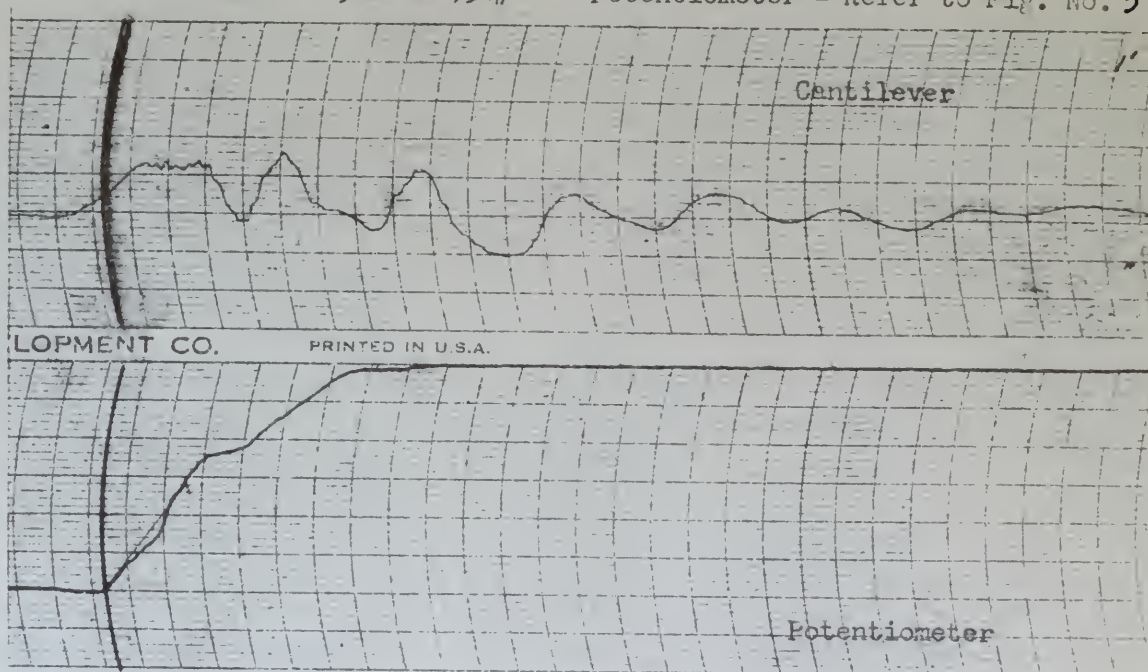


Fig. No. 8

Date: 7/10/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

Paper Speed 125 mm/sec

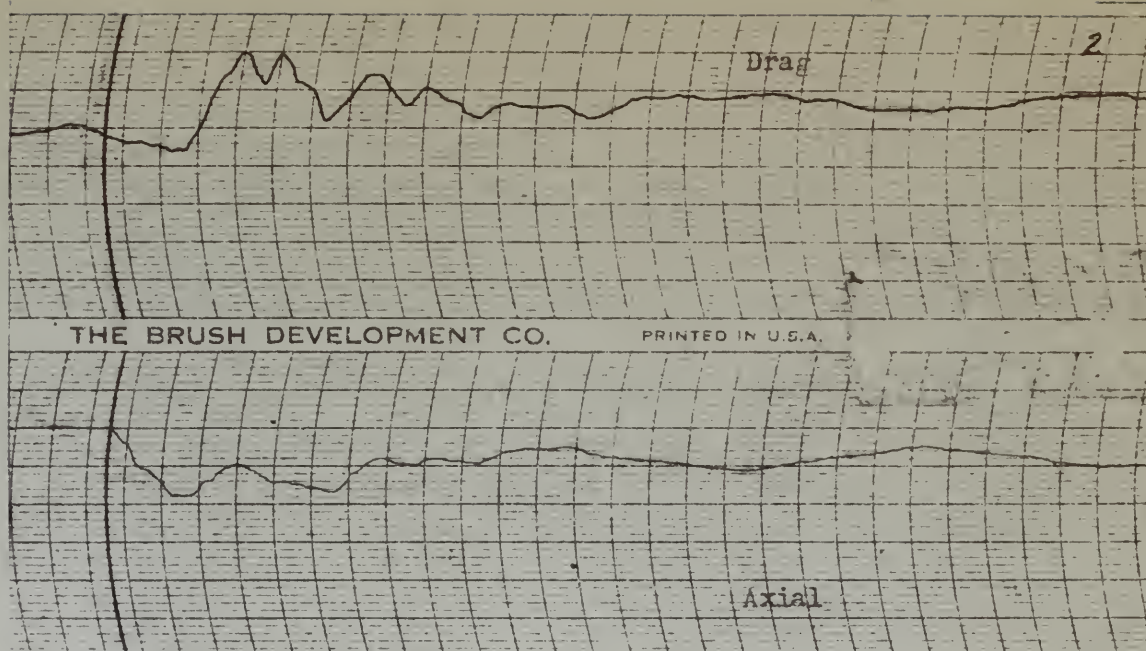
Tire Pressure - 24#

Landing Velocity - 58 FPS

Dropping Velocity - 2 FPS.







Calibration:

Drag - 1 mm = 220#

Axial - 5 mm = 930#

Cantilever - 1 mm = .375"

Potentiometer - Refer to Fig. No. 5.

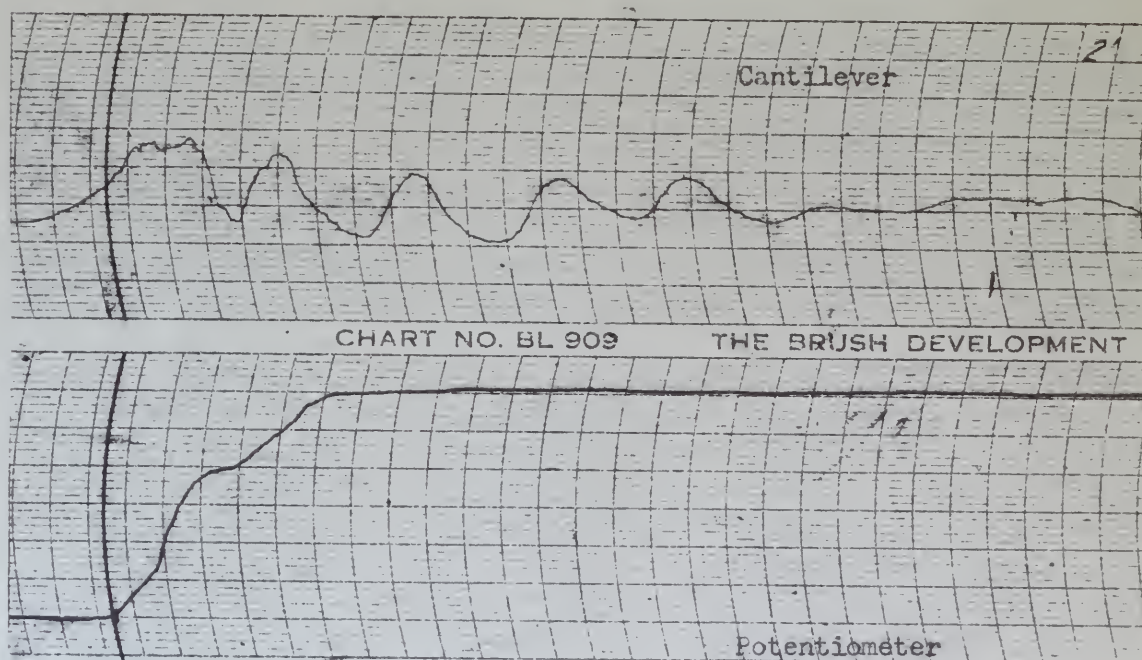


Fig. No. 9

Date: 7/10/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

Paper Speed 125 mm/sec

Tire Pressure - 24#

Landing Velocity - 58 FPS

Dropping Velocity - 3 FPS.



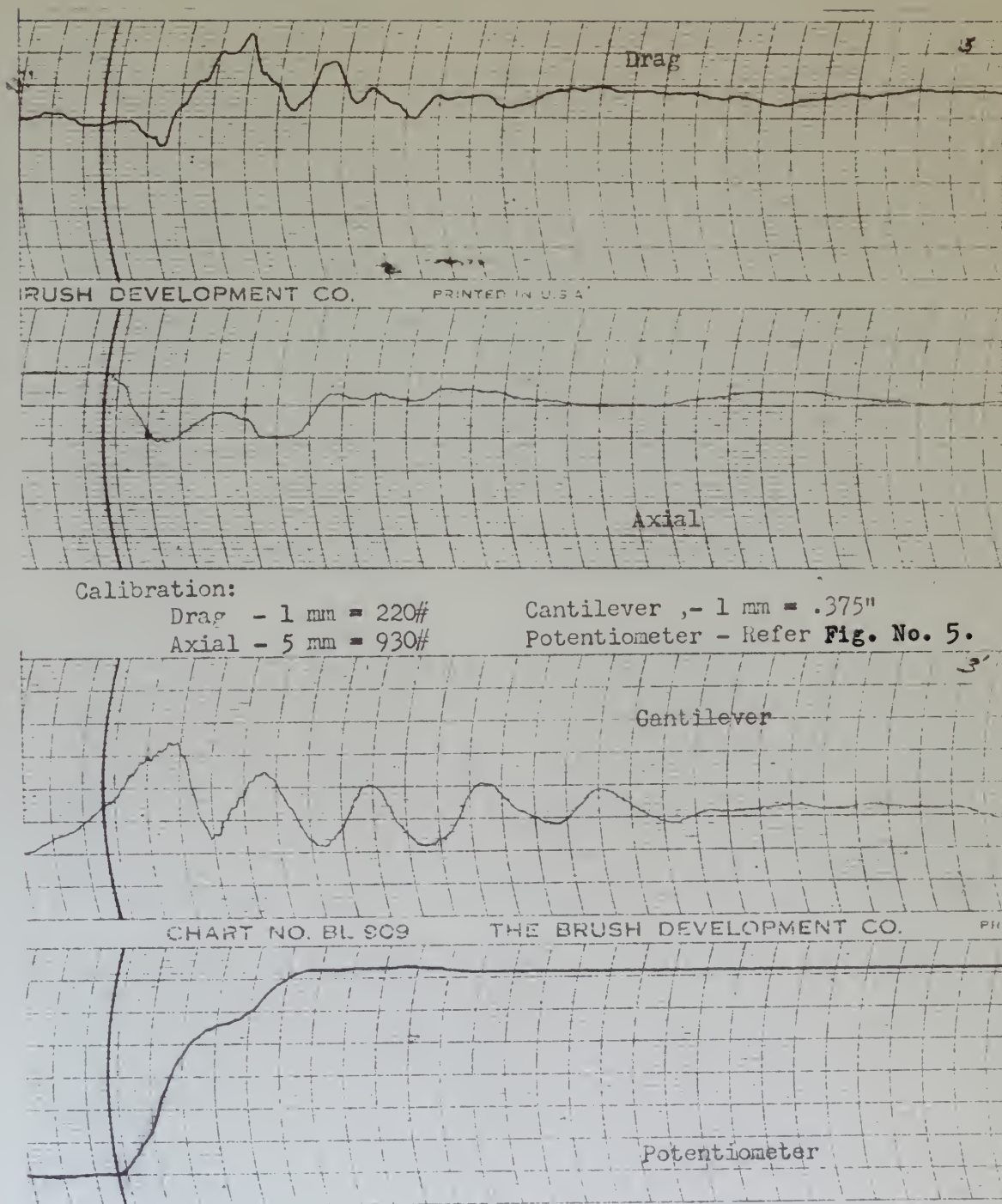


Fig. No. 10

Date: 7/10/49

Strut Angle -  $24^{\circ}$

Height - 1060#

Brush Speed 125 mm/sec

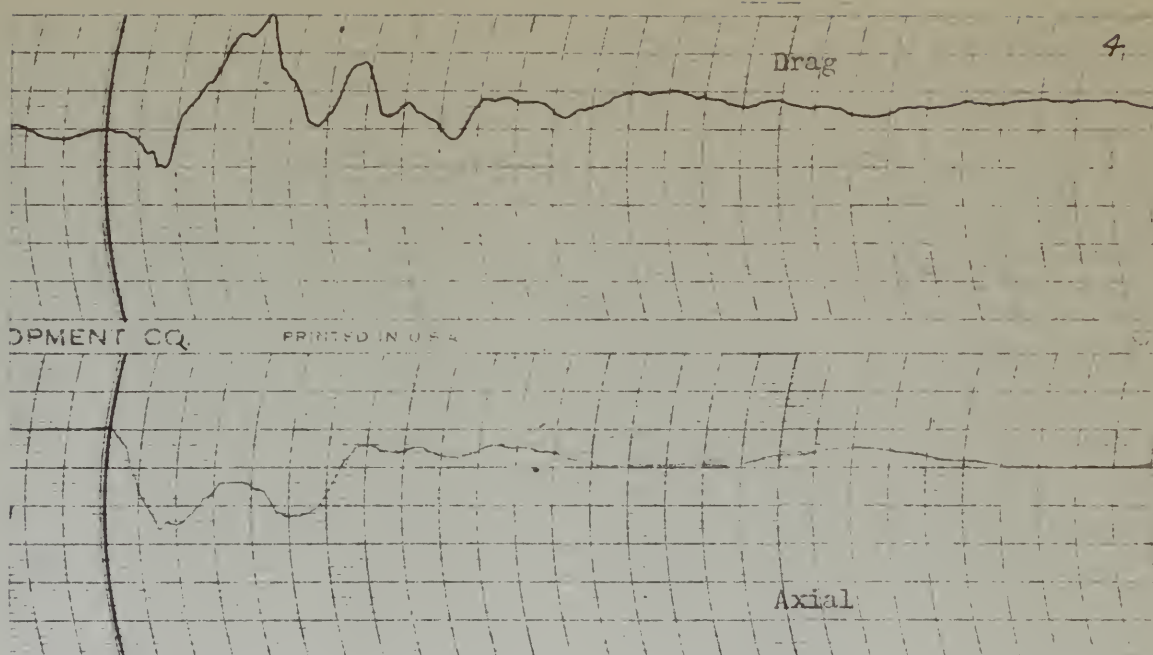
Tire Pressure - 24#

Landing Velocity - 58 FPS

Dropping Velocity - 4 FPS.







## Calibration:

Drag - 1 mm = 220#  
 Axial - 5 mm = 930#

Cantilever - 1 mm = .375"  
 Potentiometer - Refer Fig. No. 5.

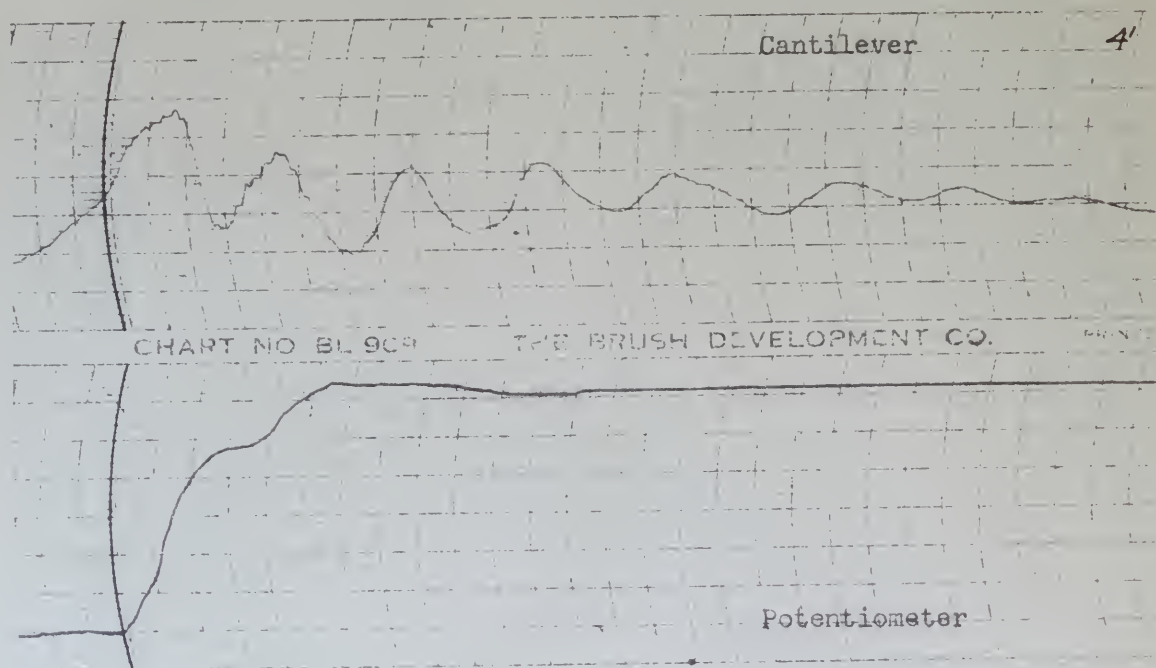


Fig. No. 11

Date: 7/10/49

Strut Angle -  $24^{\circ}$ 

Weight - 1060#

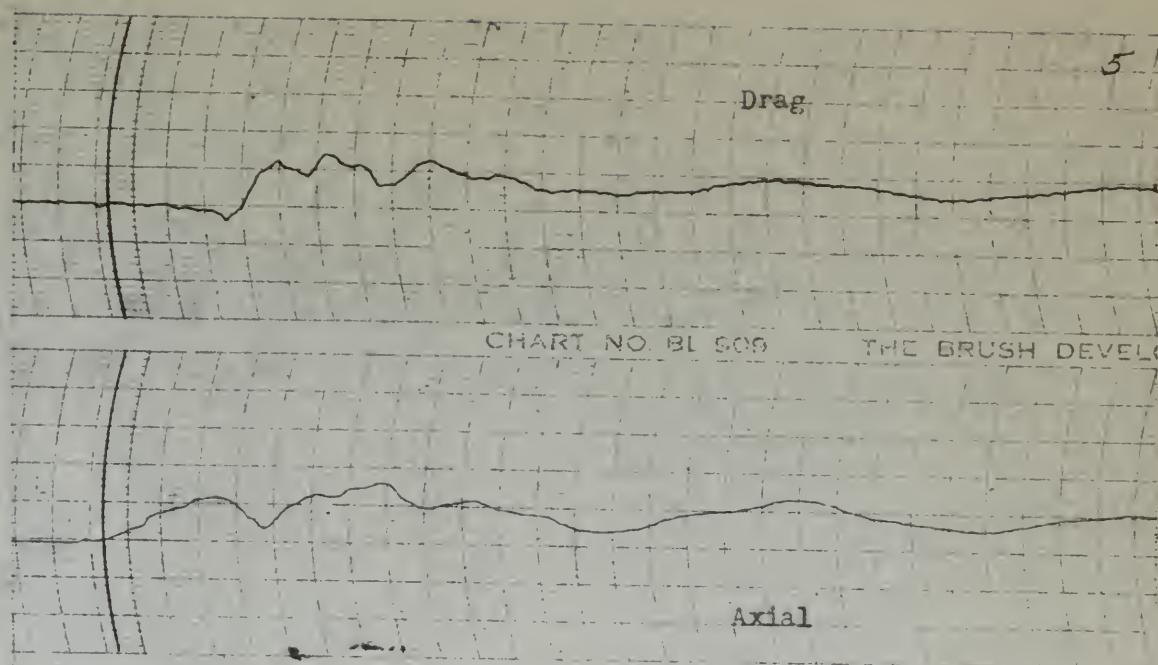
Brush Speed 125 mm/sec.

Tire Pressure - 24#

Landing Velocity - 58 FPS.

Dropping Velocity - 5 FPS.





## Calibration:

Drag - 1 mm = 110#

Cantilever - 1 mm. = .416u

Axial - 5 mm = 835#

Potentiometer - Refer to Fig. No. 5

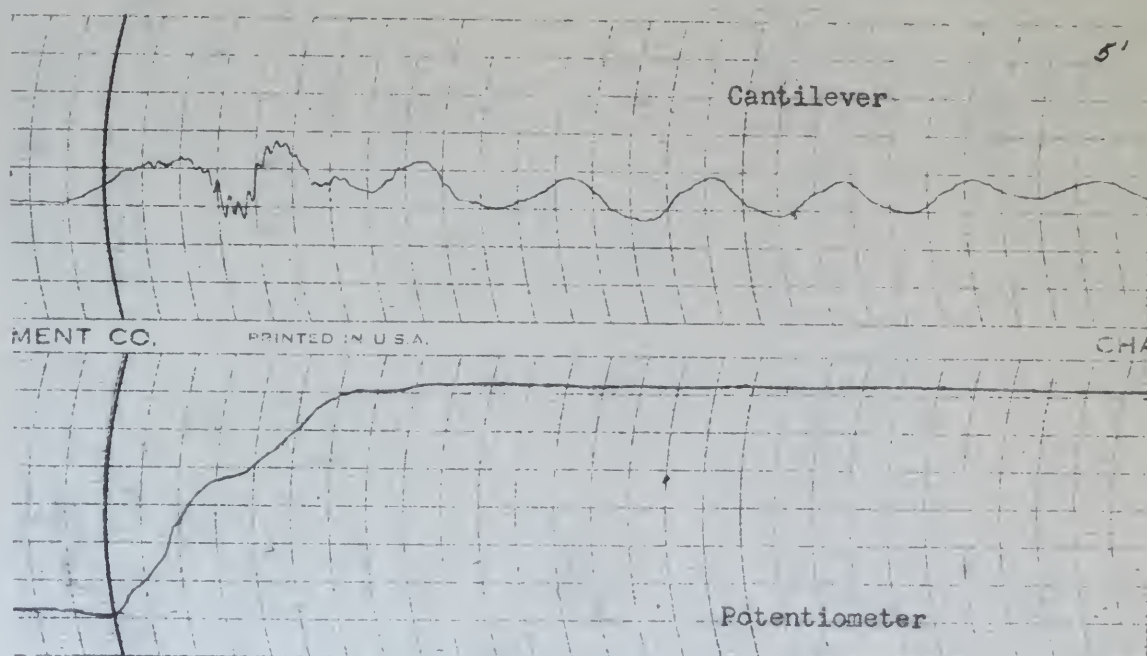


Fig. No. 12

Date: 7/13/49

Strut Angle -  $24^{\circ}$ 

Weight - 1060#

Brush Speed 125 mm/sec.

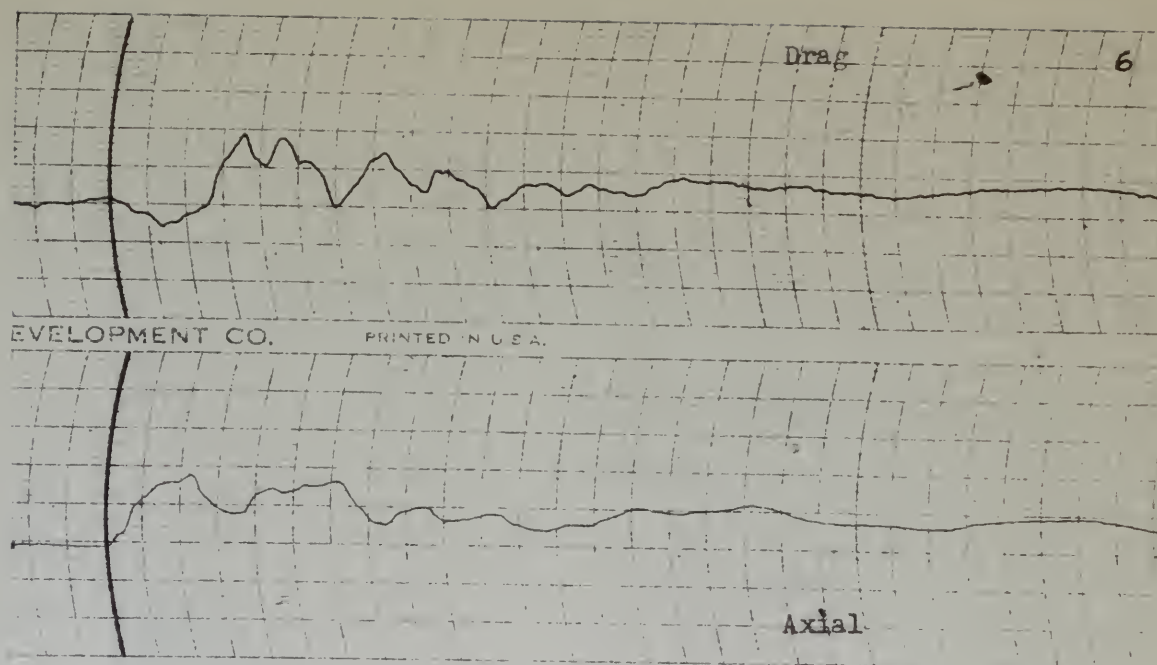
Tire Pressure - 30#

Landing Velocity - 55.5 FPS

Dropping Velocity - 2 FPS.







## Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .416"

Potentiometer - Refer to Fig. No. 5.

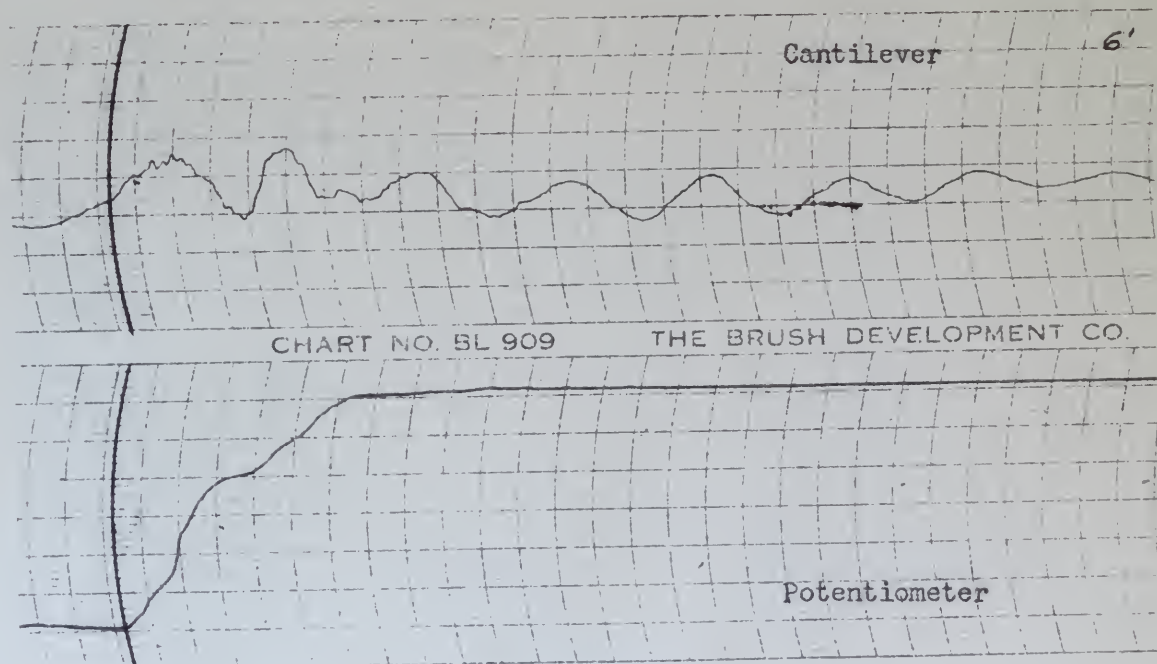


Fig. No. 13

Date: 7/13/49

Strut Angle -  $24^{\circ}$ 

Weight - 1060#

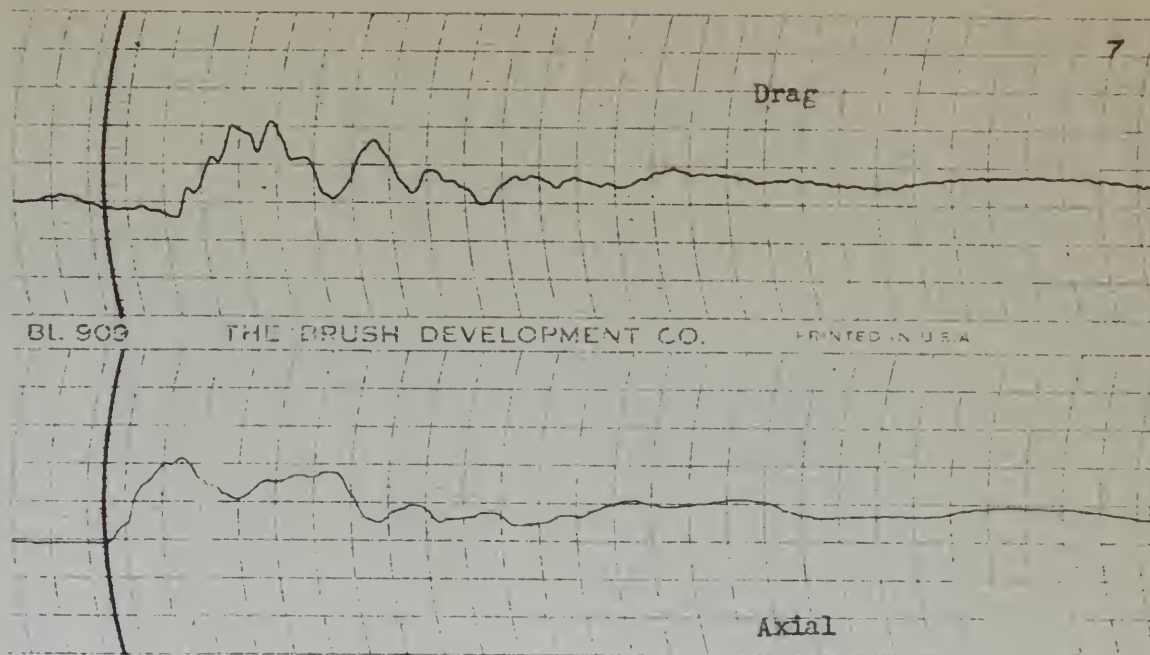
Brush Speed 125 mm/sec.

Tire Pressure - 30#

Landing Velocity - 55.5 FPS.

Dropping Velocity - 3 FPS.





## Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .416"

Potentiometer - Refer to Fig. No. 5.

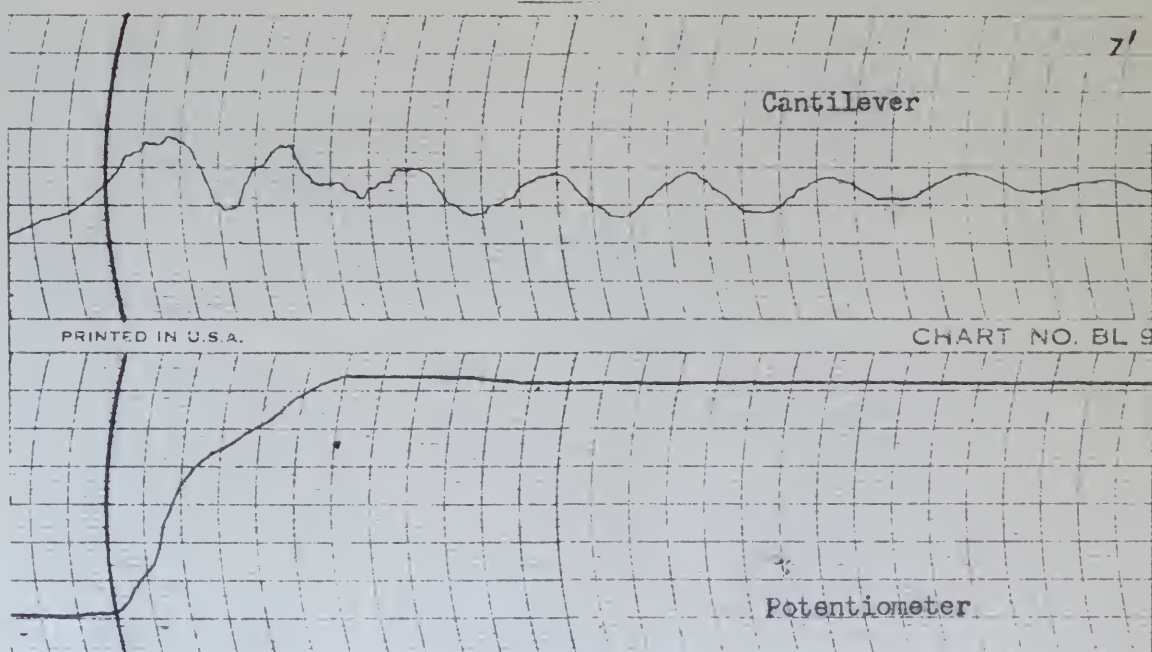


Fig. No. 14

Date: 7/13/49

Strut Angle - 24°

Weight - 1060#

Brush Speed 125 mm/sec.

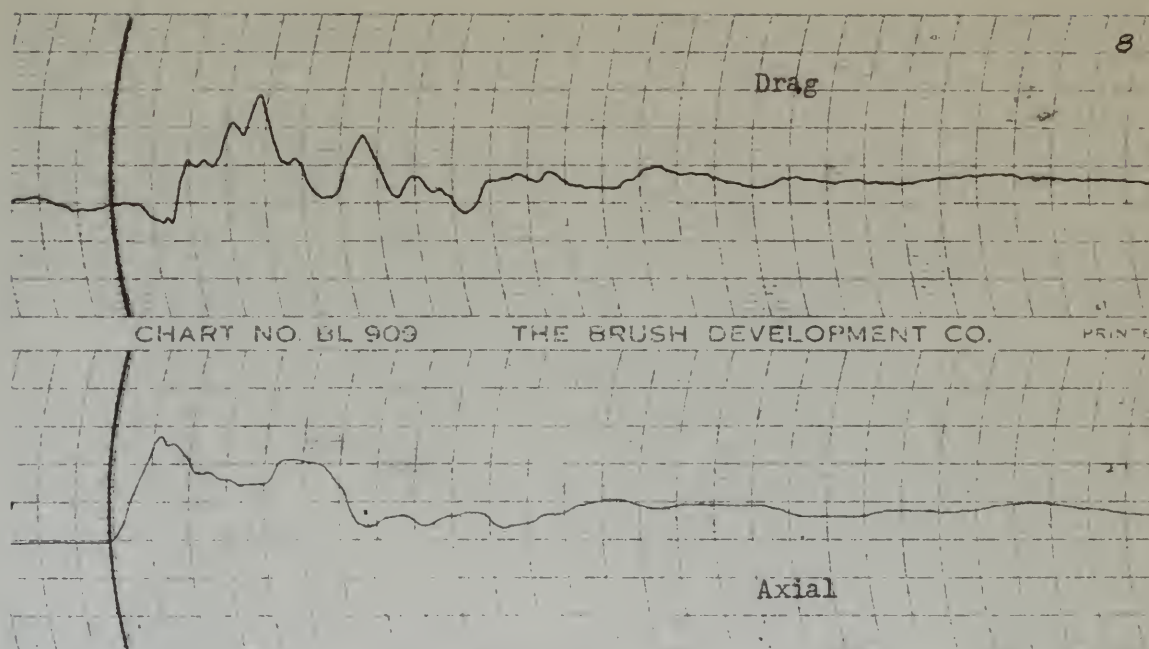
Tire Pressure - 30#

Landing Velocity - 55.5 FPS

Dropping Velocity - 4 FPS.







Calibration:

Drag - 1 mm = 110#

Cantilever - 1 mm = .416"

Axial - 5 mm = 835#

Potentiometer - Refer to Fig. No. 5.

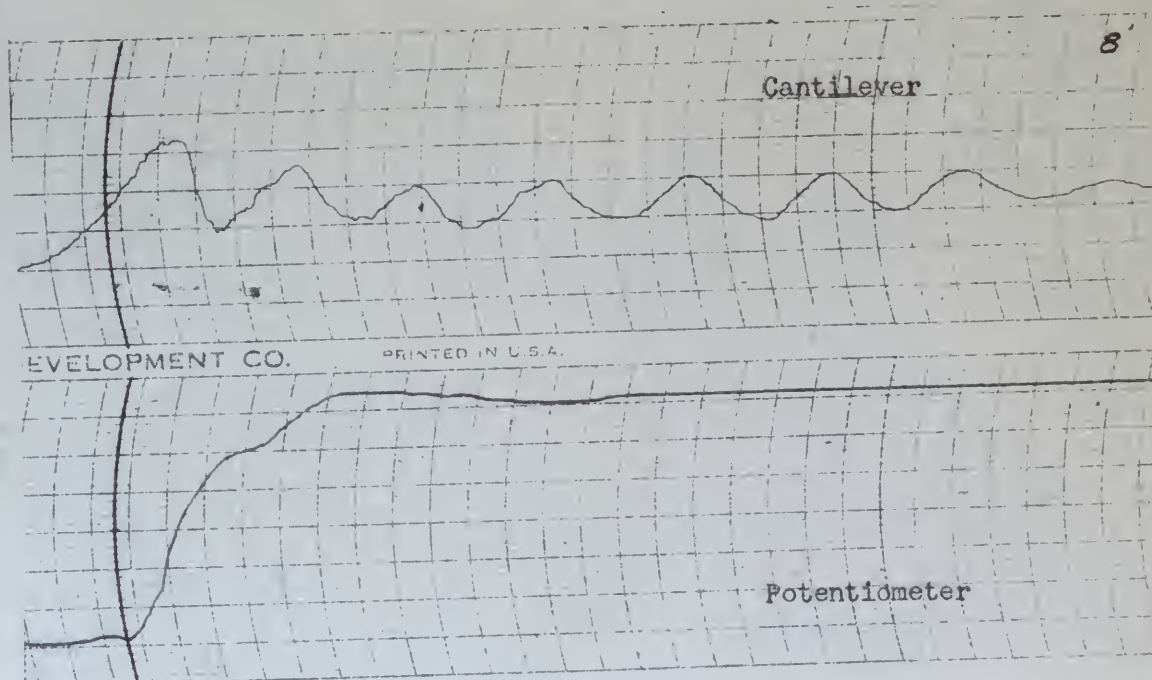


Fig. No. 15

Date: 7/13/49

Strut Angle - 24°

Weight - 1060#

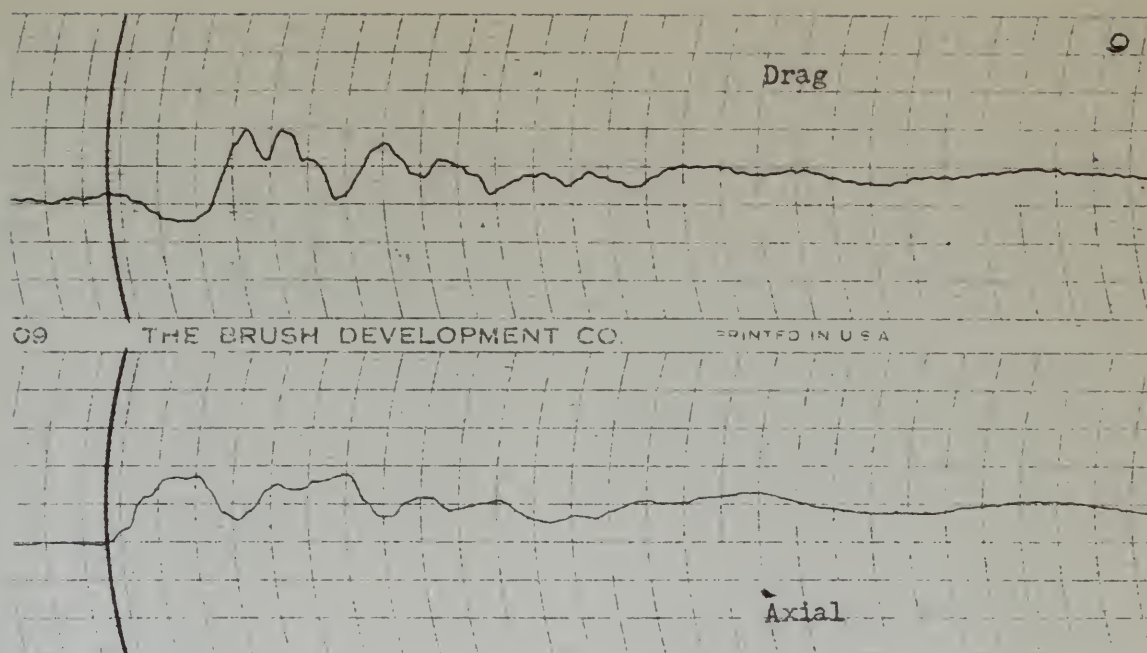
Brush Speed 125 mm/sec.

Tire Pressure - 30#

Landing Velocity - 55.5 FPS

Dropping Velocity - 5 FPS.





Calibration:

Drag - 1 mm = 110#

Cantilever - 1 mm = .416 "

Axial - 5 mm = 835#

Potentiometer - Refer to Fig. No. 5.

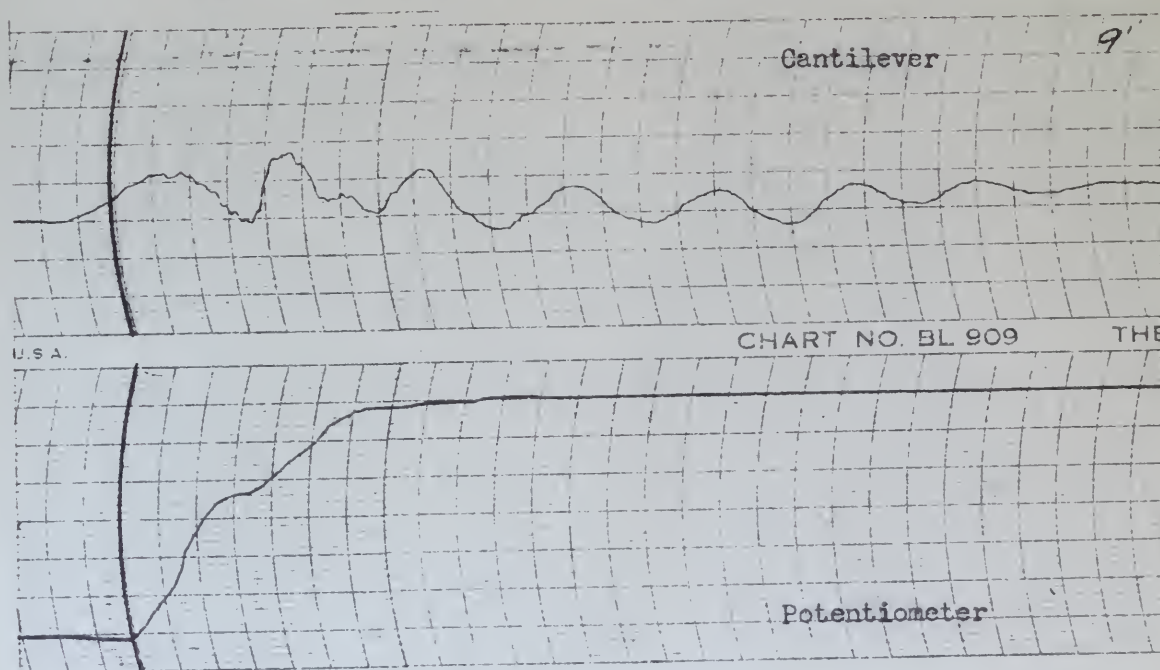


Fig. No. 16

Date: 7/13/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

Brush Speed 125 mm/sec

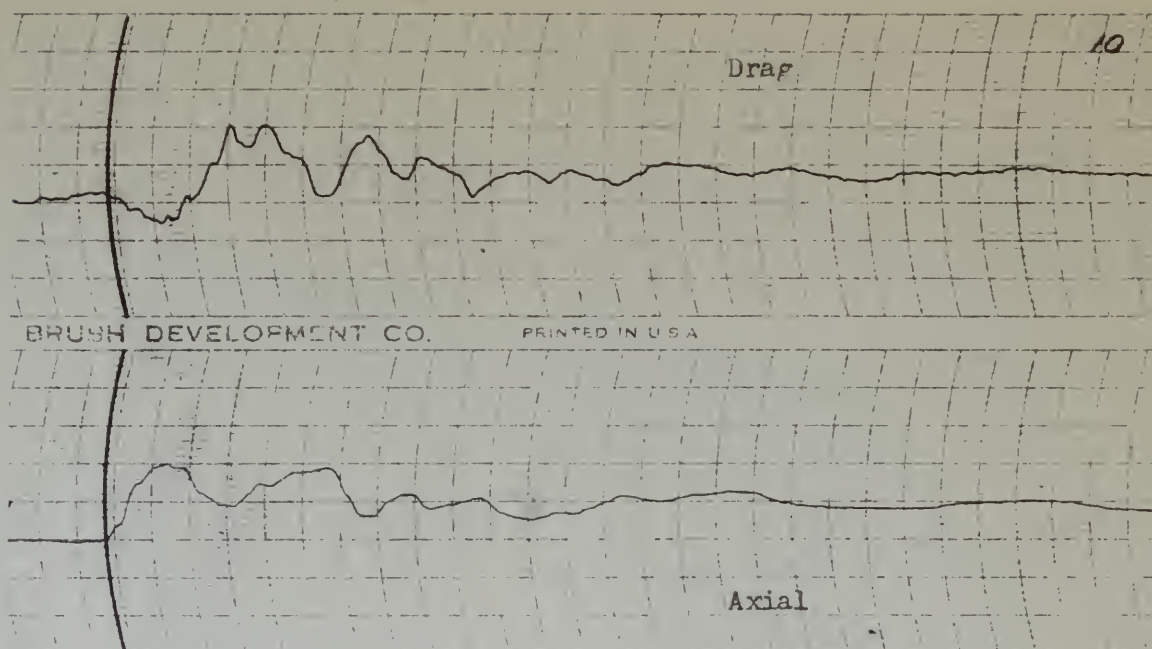
Tire Pressure 35#

Landing Velocity - 56 FPS

Dropping Velocity - 2 FPS.







Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .416"

Potentiometer - Refer to Fig. No. 5.

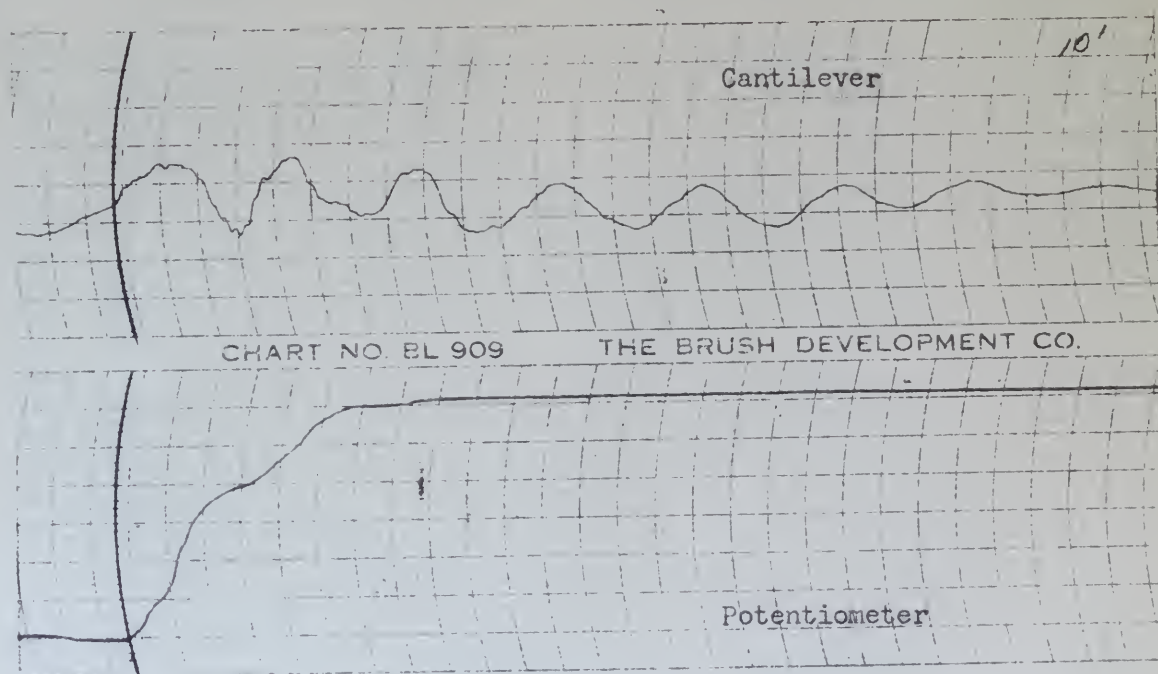


Fig. No. 17

Date: 7/13/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

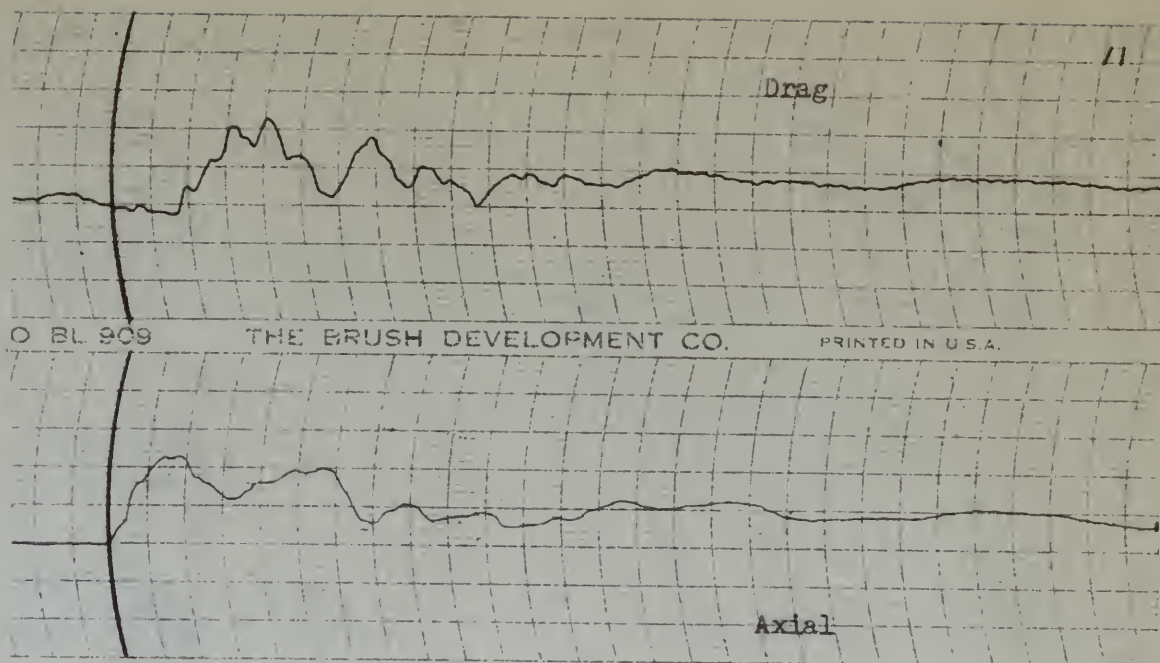
Brush Speed 125 mm/sec.

Tire Pressure 35#

Landing Velocity - 56 FPS

Dropping Velocity - 3 FPS.





## Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .416"

Potentiometer - Refer to Fig. No. 5.

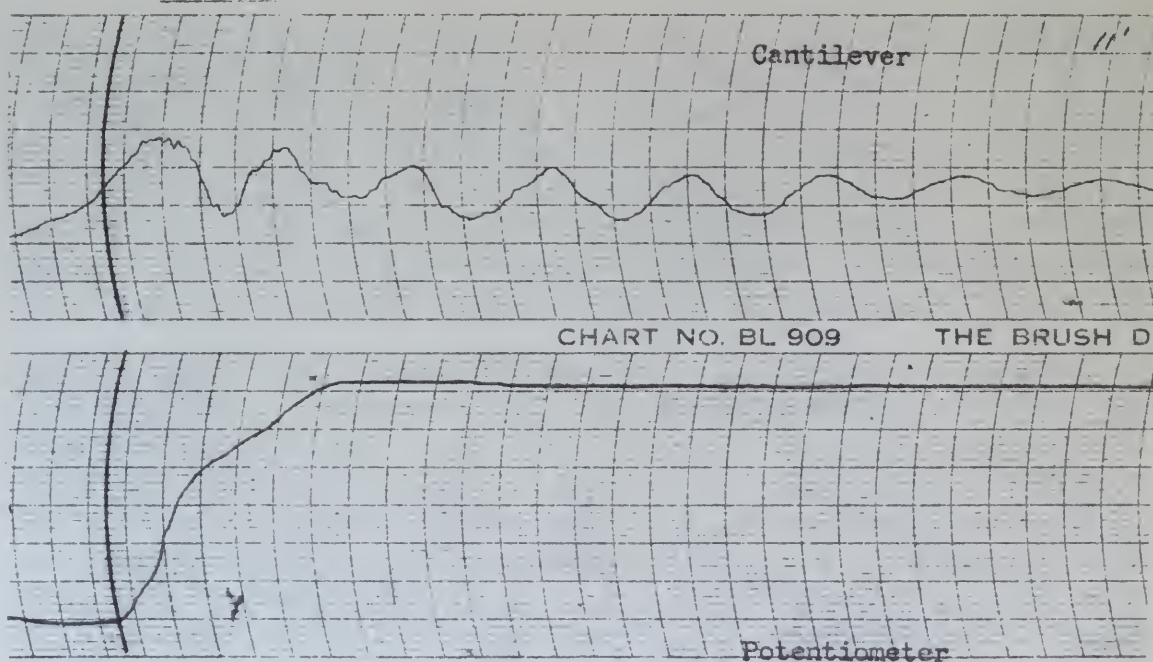


Fig. No. 18

Date: 7/13/49

Strut Angle  $-24^{\circ}$ 

Weight - 1060#

Brush Speed 125 mm/sec.

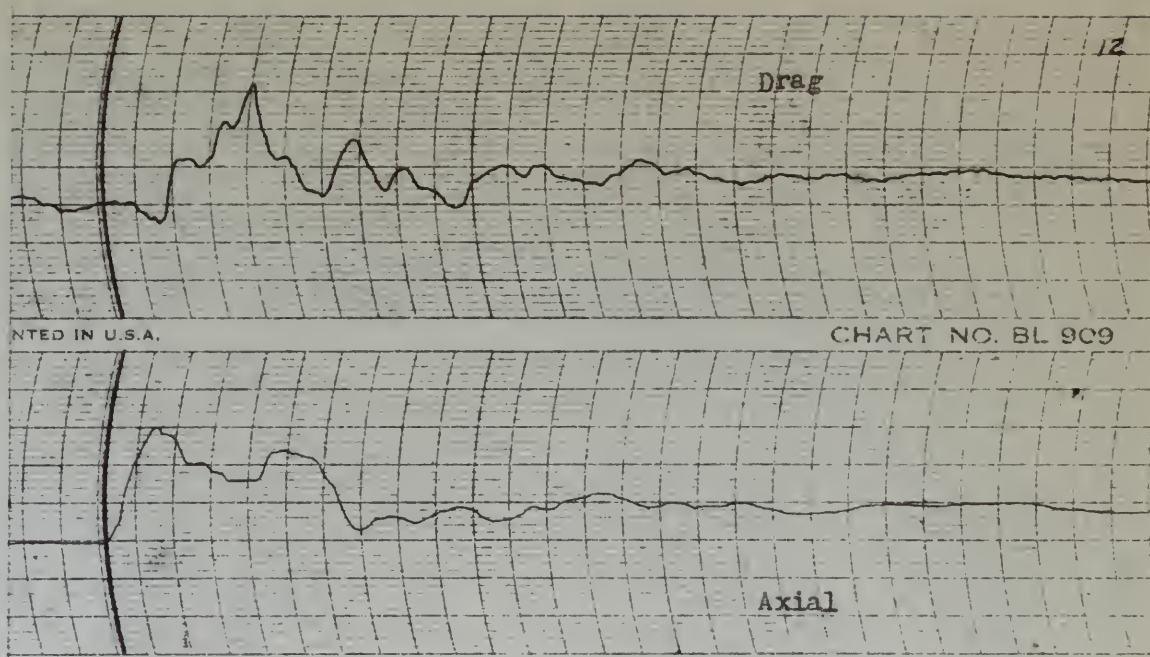
Tire Pressure - 35#

Landing Velocity - 56 FPS

Dropping Velocity - 4 FPS.







Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .416"

Potentiometer - Refer to Fig. No. 5.

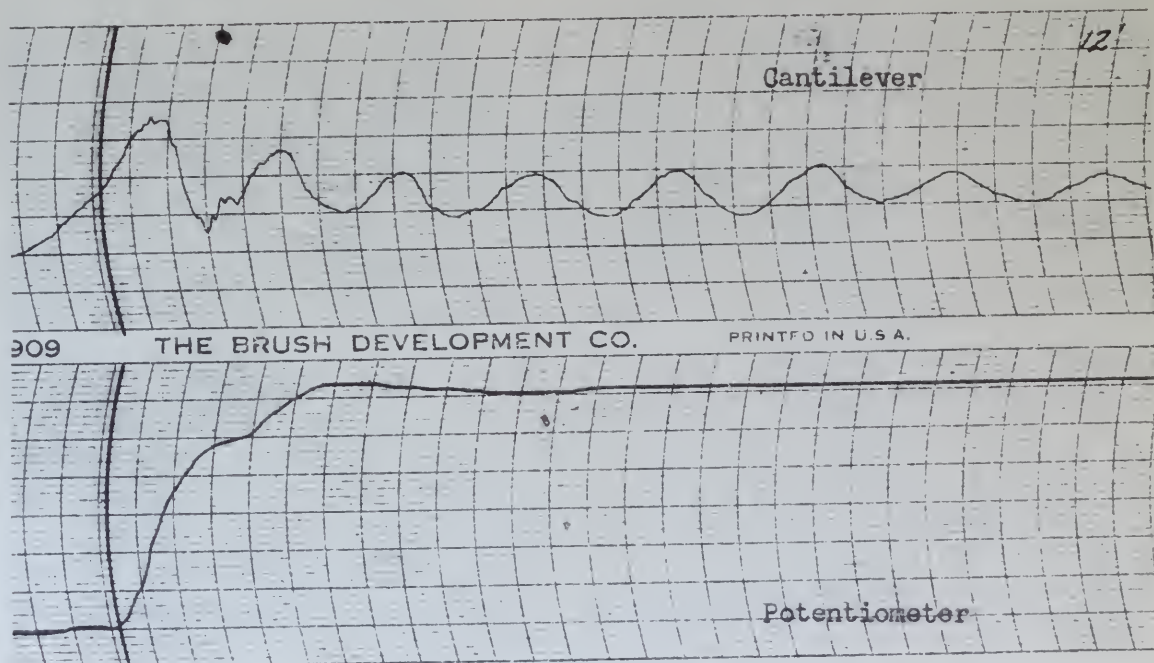


Fig. No. 19

Date: 7/13/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

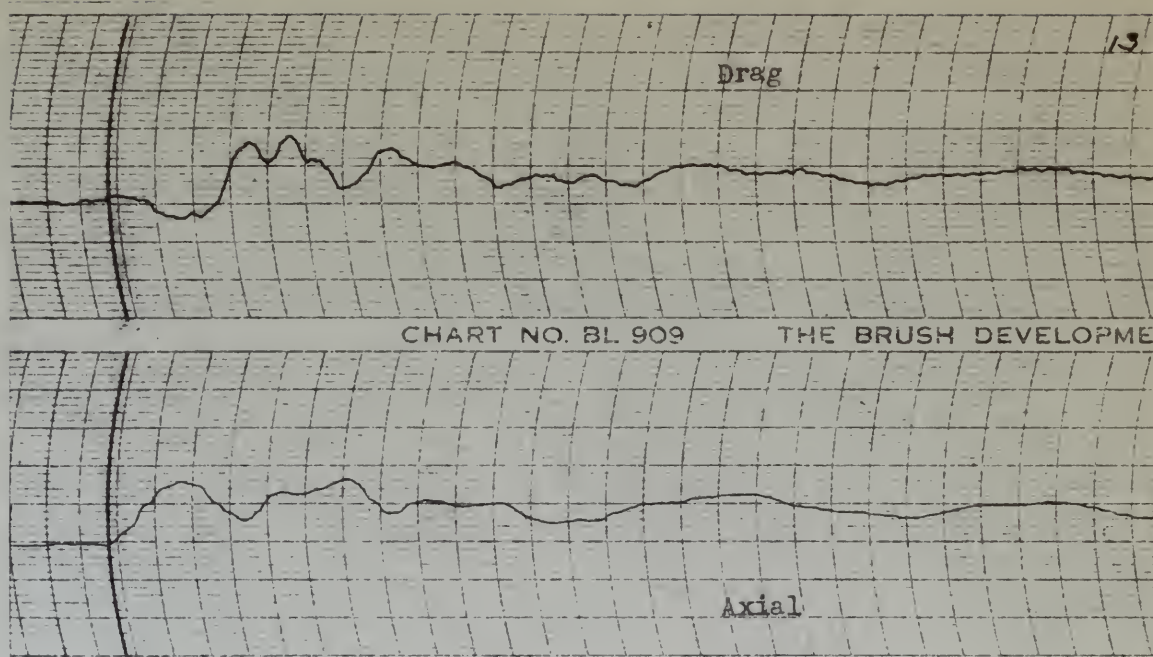
Brush Speed 125 mm/sec.

Tire Pressure - 35#

Landing Velocity - 56 FPS

Dropping Velocity - 5 FPS.





## Calibration:

Drag - 1 mm = 110#  
 Axial - 5 mm = 835#

Cantilever-1 mm = .208"  
 Potentiometer - Refer to Fig. No. 5

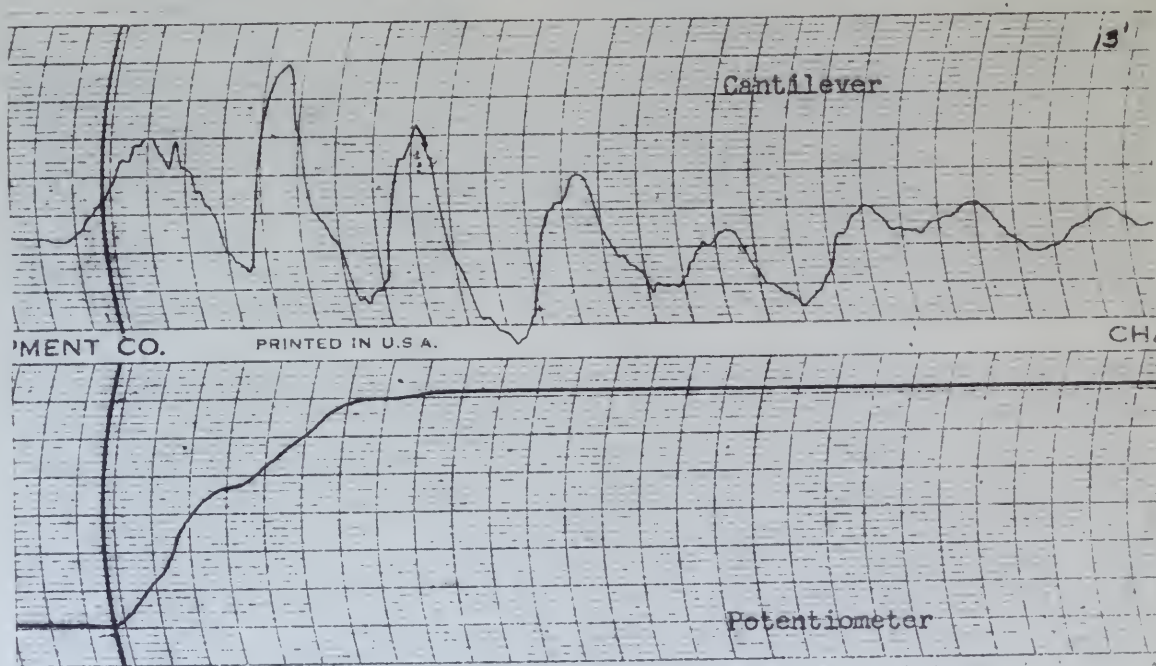


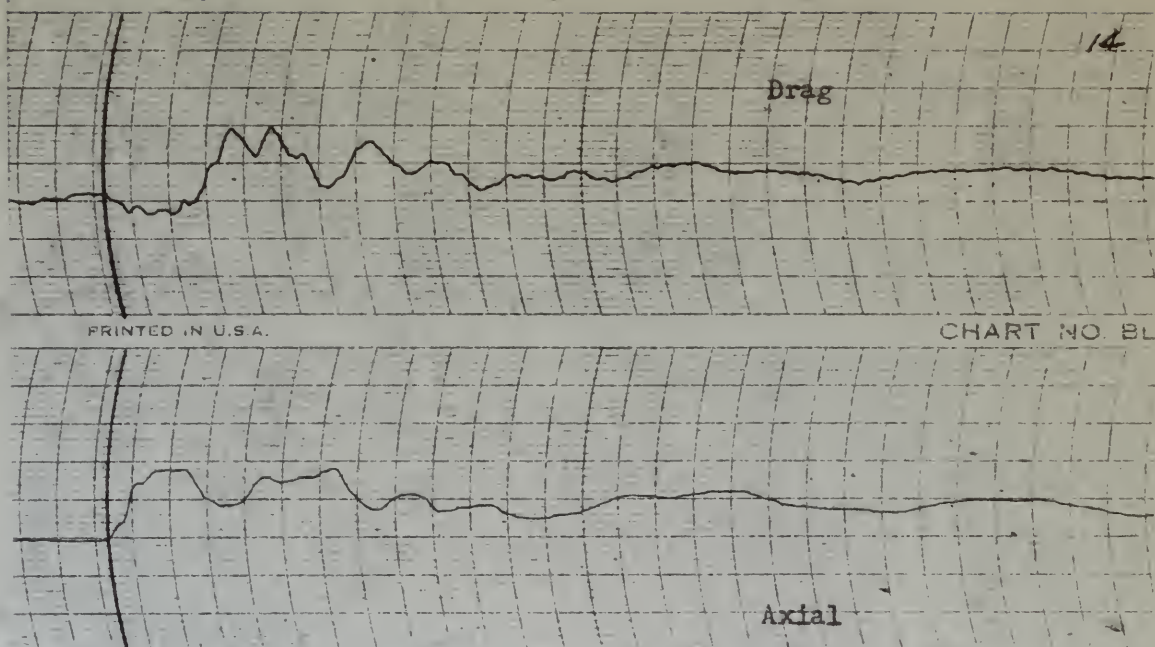
Fig. No. 20

Date: 7/13/49  
 Strut Angle - 24°  
 Weight - 1060#  
 Brush Speed 125 mm/sec.

Tire Pressure - 40#  
 Landing Velocity - 56 FPS.  
 Dropping Velocity - 2 FPS.







Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .208"

Potentiometer - Refer to Fig. No. 5.

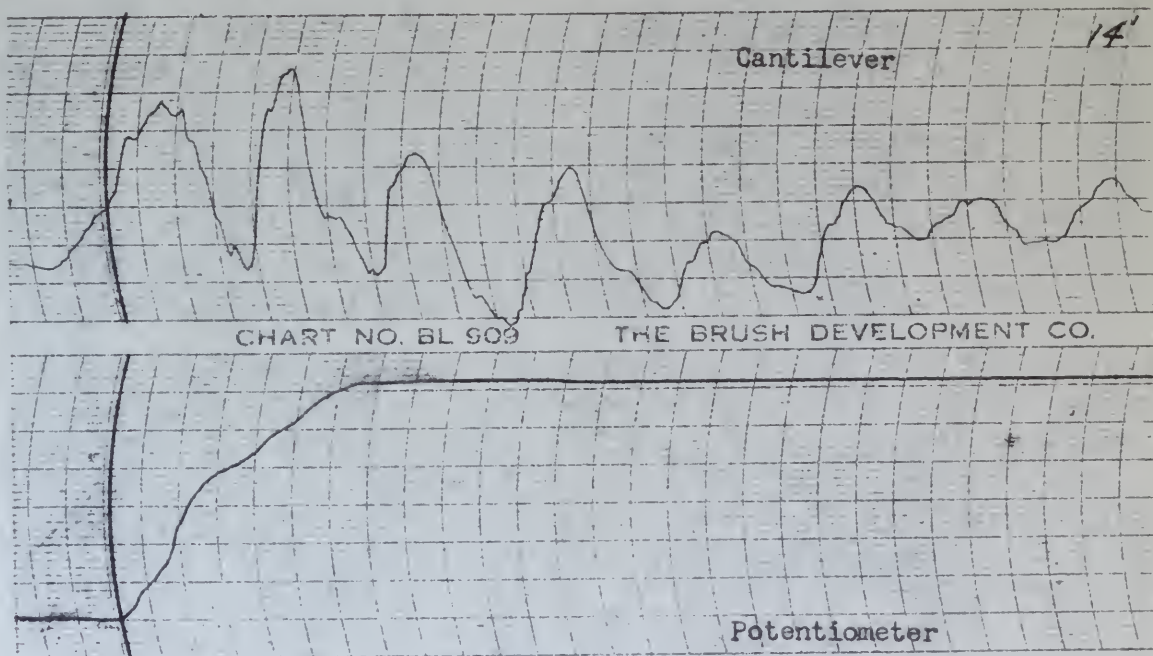


Fig. No. 21

Date: 7/13/49

Strut Angle -  $24^{\circ}$

Weight - 1060#

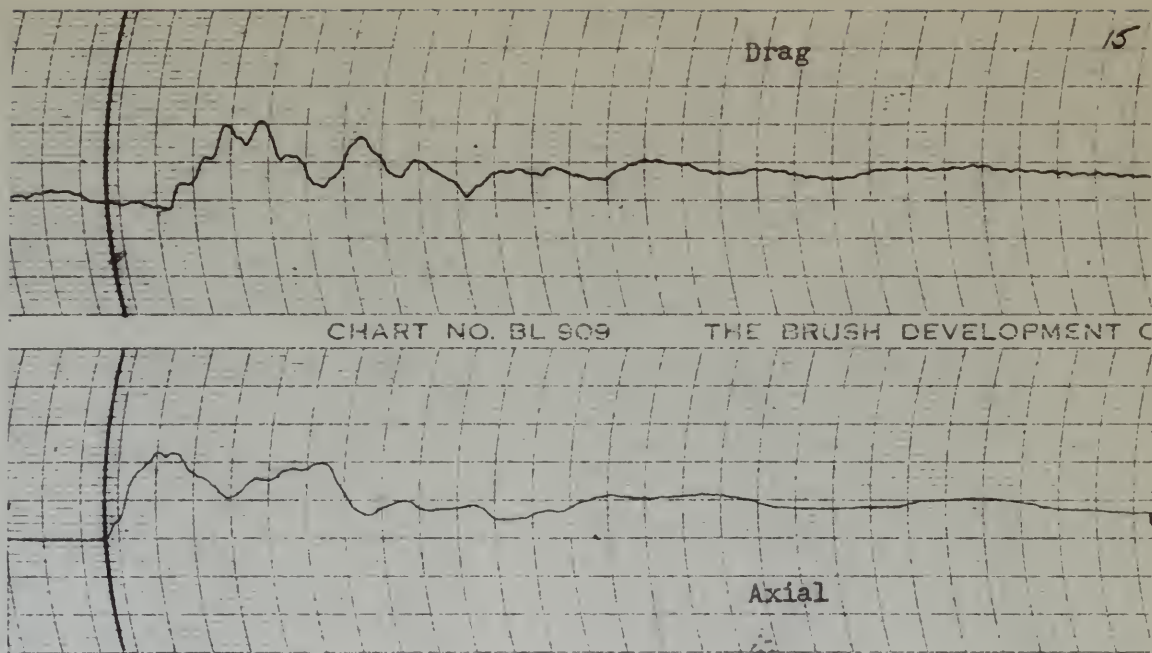
Brush Speed 125 mm/sec

Tire Pressure - 40#

Landing Velocity - 56 FPS.

Dropping Velocity - 3 FPS.





## Calibration:

Drag - 1 mm = 110#

Axial - 5 mm = 835#

Cantilever - 1 mm = .208"

Potentiometer - Refer to Fig. No. 5.

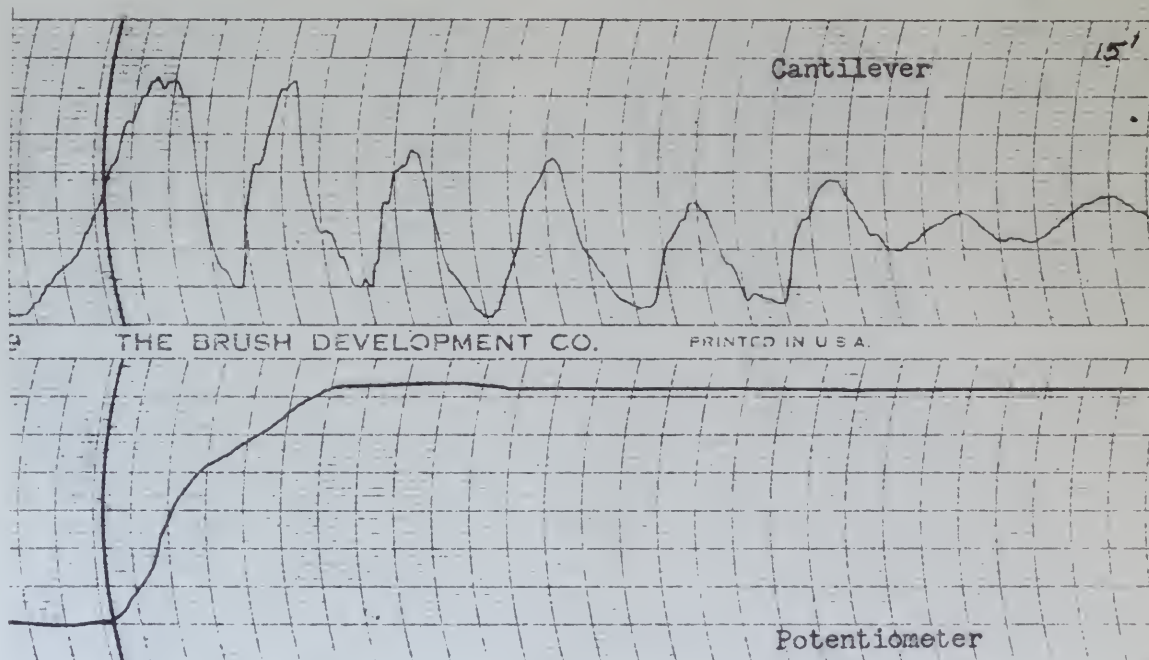


Fig. No. 22

Date: 7/13/49

Strut Angle -  $24^{\circ}$ 

Weight - 1060#

Brush Speed 125 mm/sec.

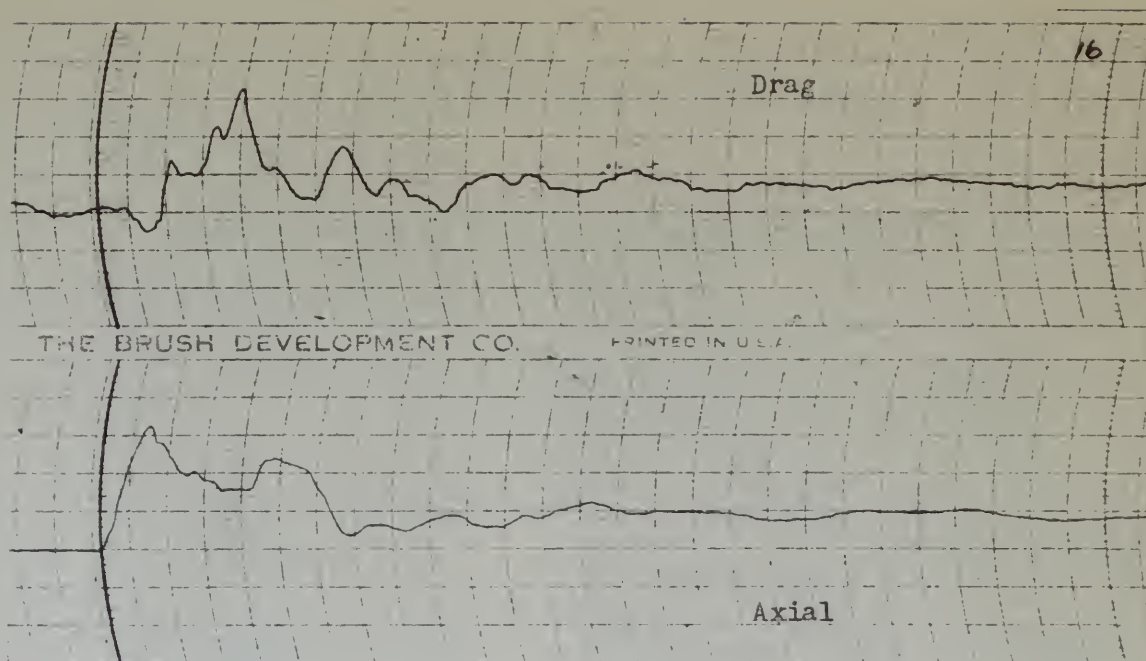
Tire Pressure - 40#

Landing Velocity - 56 FPS.

Dropping Velocity - 4 FPS.







## Calibration:

Drag - 1 mm = 110#  
 Axial - 5 mm = 835#

Cantilever - 1 mm = .208"  
 Potentiometer - Refer to Fig. No. 5.



Fig. No. 23

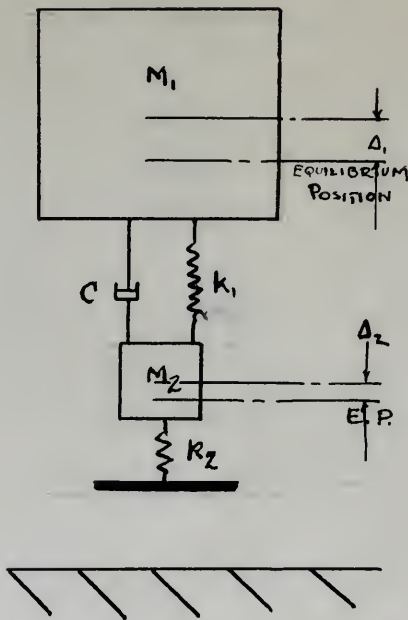
Date: 7/13/49  
 Strut Angle -  $24^{\circ}$   
 Weight - 1060#  
 Brush Speed 125 mm/sec.

Tire Pressure - 40#  
 Landing Velocity - 56 FPS.  
 Dropping Velocity - 5 FPS.



## COMPARISON OF THE TEST AND THEORY

The following problem incorporates the constants computed into the theory developed.



$$w_1 = 939 \text{ \#}$$

$$w_2 = 121 \text{ \#}$$

$$M_1 = 29.16 \text{ \#sec.}^2/\text{ft.}$$

$$M_2 = 3.76 \text{ \#sec.}^2/\text{ft.}$$

$$k_{\text{effective}} = k_{\text{measured}} \cos 24^\circ$$

$$k_1 = 1584 \text{ \#/ft.}$$

$$k_2 = 11280 \text{ \#/ft.}$$

$$-\Delta_1 = -.094 \text{ ft.}$$

$$k_2 = 12300 \text{ \#/ft.}$$

$$-\Delta_2 = -.636 \text{ ft.}$$

### DETERMINATION OF (C) DAMPING CONSTANT

Example:

From Fig. No. 14

$$\dot{x} = \frac{dx}{dt} = \frac{1.07}{.04} = 26.8 \text{ ft./sec.}$$

at  $t = .04 \text{ sec.}$

$$R_1 = 69 \text{ \#/in.}$$

$$F = \frac{2.5}{5} \times 835 \text{ \#} = 1615 \text{ \#}$$

Oleo Deflection = 1.07 inches

$$F = C\dot{x} + k_1 x$$

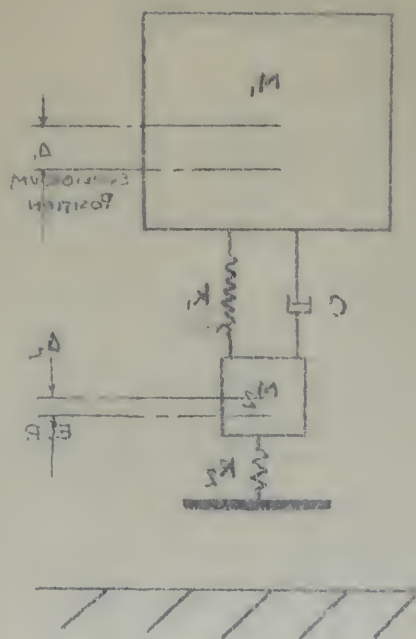
$$1615 = C(26.8) + 69 \times 1.07$$

$$C = \frac{1549}{26.8} = 57.5 \text{ \#sec./in.}$$

$$C = 688 \text{ \#sec./ft.}$$

# PROBLEM 10.10

The following problem involves the analysis of a system of two masses and two springs.



$$\begin{aligned}
 M_1 &= 10 \text{ kg} \\
 M_2 &= 5 \text{ kg} \\
 K_1 &= 200 \text{ N/m} \\
 K_2 &= 100 \text{ N/m} \\
 C &= 10 \text{ Ns/m} \\
 \Delta_1 &= 0.1 \text{ m} \\
 \Delta_2 &= 0.05 \text{ m}
 \end{aligned}$$

Find the natural frequency and the damping ratio of the system.

Solution:

$$\begin{aligned}
 \omega_n &= \sqrt{\frac{K_1 + K_2}{M_2}} = \sqrt{\frac{200 + 100}{5}} = 10 \text{ rad/s} \\
 \zeta &= \frac{C}{2\sqrt{K_1 + K_2} M_2} = \frac{10}{2\sqrt{300} \cdot 5} = 0.1732
 \end{aligned}$$



$$C_{ave.} = \underline{\underline{653}}$$

### Equations of Motion

$$-M_1 \ddot{x}_1 - C(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

$$-M_2 \ddot{x}_2 - k_2 x_2 + k_1(x_1 - x_2) + C(\dot{x}_1 - \dot{x}_2) = 0$$

Or

$$s^4 + 196s^3 + 3,475.65s^2 + 67,182.05s + 162,965.34 = 0$$

### By Synthetic Division

$$s_1 = -2.759$$

$$s_2 = 178.62$$

$$s_3 = -7.31 - i 16.65$$

$$s_4 = -7.31 + i 16.65$$

$$A = \phi_1 A'$$

$$A = -50.03A'$$

$$B = \phi_2 B'$$

$$B = -.141B'$$

$$C = \phi_3 C'$$

$$C = .663 + i .862$$

$$D = \phi_4 D'$$

$$D = .663 - i .862$$

$$A' = \frac{E_1}{E}; B' = \frac{E_2}{E}; C' = \frac{E_3}{E}; D' = \frac{E_4}{E}$$

$$E = 1 286,359.82;$$

$$A' = .00745;$$

$$E_1 = 1 2133.98;$$

$$B' = -.01699;$$

$$E_2 = -1 4866.03;$$

$$C' = -.052 + i .112;$$

$$E_3 = -32.145 - i 14.91$$

$$D' = -.052 - i .112;$$

$$E_4 = +32.145 - i 14.91$$

$$A = -.373$$

$$B = .0024;$$

Equation of motion

$$0 = (g_1 - g_2) \ddot{x} - (g_1 \ddot{y}_1 + g_2 \ddot{y}_2) - \ddot{y}_1 \ddot{y}_2$$

$$0 = (g_1 - g_2) \ddot{x} + (g_1 \ddot{y}_1 + g_2 \ddot{y}_2) - \ddot{y}_1 \ddot{y}_2$$

or

$$g_1 \ddot{y}_1 \ddot{y}_2 + g_2 \ddot{y}_1 \ddot{y}_2 + g_1 \ddot{y}_1 \ddot{y}_2 + g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$0 =$$

of the motion

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_1 \ddot{y}_1 \ddot{y}_2 = 0$$

$$g_2 \ddot{y}_1 \ddot{y}_2 = 0$$

$$C = -.131 + i .029;$$

$$D = -.131 - i .029;$$

$$x_1 = -.373e^{-2.759t} + .0024e^{-178.62t} + e^{-7.31t} (-.262 \cos. 16.65t + .058 \sin 16.65t)$$

$$x_2 = .00745e^{-2759t} - .01699e^{-178.62t} + e^{-7.31t} (-.104 \cos. 16.65t + .224 \sin 16.65t)$$

Frequency,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{16.65} = \frac{4.09}{2\pi} = .651 \text{ CPS}$$

Period of Vibration,

$$T = \frac{2\pi}{\omega} = \frac{6.28}{16.65} = .377 \text{ sec.}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log L_n = 0$$

$$19001. \quad 2 = 17 \cdot 2 = 34$$

$$120.00 \text{ m}^2 \times 100 \text{ m}^2 + 120.00 \text{ m}^2 \times 100 \text{ m}^2 = 24000.00 \text{ m}^2$$

$$201.7 = 20.7 \times 10 + 220.075 - 220.075 \times 20.7 / 100 = 58$$

$$\text{KVO } 124. = \frac{20.1}{115} = 0.1747 \sqrt{\frac{1}{11}} = \frac{0.1}{115} = 0$$

$$\cos \varphi_{\text{н}} = \frac{R_{\text{н}}}{Z_{\text{н}}} = \frac{\pi}{\omega} = 1$$



There is a close agreement between the test data and the theory. The computed period of vibration is .377 seconds whereas the experimental period is .388 seconds. The theory is within 2.8 per cent of the experimental values. The theoretical maximum force for the landing gear's dropping velocity of four feet per second shows an axial force of 2440 pounds. An experimental force, 1910 pounds, was indicated on the Brush recorder. The association between these two values is not as close as that of the period previously mentioned. The affinity between the maximum theoretical force is 22 per cent greater than in the experimental axial load.

From Fig. Nos. 6 & 7 are chosen the values of spring constant used in the illustrated problem. These values are not true constants, but are assumed to be in order to simplify computations. For example, as the air compressed in the oleo, the force versus distance curve was not of a linear relation. This was also true for the tire. In other words, these "spring constants" are not in reality, constant. Had they been a true constant the agreement between the theoretical and experimental would have been closer.

There is a close agreement between the last data and the theory. The computed period of vibration is .377 seconds whereas the experimental period is .368 seconds. The theory is within 2.5 per cent of the experimental value. The theoretical maximum force for the leading member's flexing velocity of 100 feet per second shows an axial force of 124,000 pounds. An experimental force, 121,000 pounds, was indicated on the strain transducer. The correlation between these two values is not as close as that of the period previously mentioned. The difference between the maximum theoretical force is 2.5 per cent greater than in the experimental value.

From Fig. 50, 51 & 52 are shown the values of spring constant used in the illustrated problem. These values are not true constants, but are assumed to be in order to simplify calculations. For example, as the air pressure in the pipe, the force versus distance curve was not of a linear relation. This was also true for the pipe. In other words, these "spring constants" are not in reality, constants. The only thing a true constant is the agreement between the theoretical and experimental results have been shown.

$x_2 = .007 e^{-2.759t} - .017 e^{-178.62t} + e^{-7.31t} (-.104 \cos 16.65t + .224 \sin 16.65t)$						
t	(c) $\cos \omega$	$\sin \omega$	(E) $e^{-7.31t}$	(F) $+ .224D$	(E+F)	$x_2$
.0	1	0	-.1040	0	-.1040	-.1040
.01	.986	.165	-.1036	.0370	-.0666	-.0656
.02	.945	.326	-.0982	.0730	-.0252	-.0218
.03	.878	.479	-.0912	.1072	+.0160	+.0128
.04	.786	.617	-.0818	.1382	+.0564	+.0422
.05	.675	.738	-.0702	.1654	+.0952	+.0661
.06	.540	.841	-.0561	.1886	+.1322	+.0852
.07	.390	.921	-.0406	.2062	+.1652	+.0992
.08	.239	.971	-.0205	.2180	+.1875	+.1043
.09	.071	.997	-.0073	.2235	+.2162	+.1118
.10	-.091	.996	+.0095	.2235	+.2320	+.1117
.11	-.257	.967	+.0278	.2170	+.2448	+.1095
.12	-.416	.909	+.0432	.2035	+.2467	+.1025
.13	-.555	.830	+.0576	.1860	+.2436	+.0941
.14	-.690	.725	+.0716	.1625	+.2341	+.0844
.15	-.801	.598	+.0833	.1340	+.2173	+.0731
.16	-.887	.468	+.0922	.1050	+.1972	+.0611
.17	-.925	.302	+.0990	.0681	+.1671	+.0483
.18	-.990	.140	+.1030	.0314	+.1344	+.0347
.19	-.999	-.018	+.1039	-.0040	+.0999	+.0249
.20	-.984	-.190	+.1024	-.0425	+.0599	+.0139
.21	-.936	-.345	+.0972	-.0786	+.0186	+.0043
.22	-.862	-.494	+.0900	-.1110	-.0210	-.0042







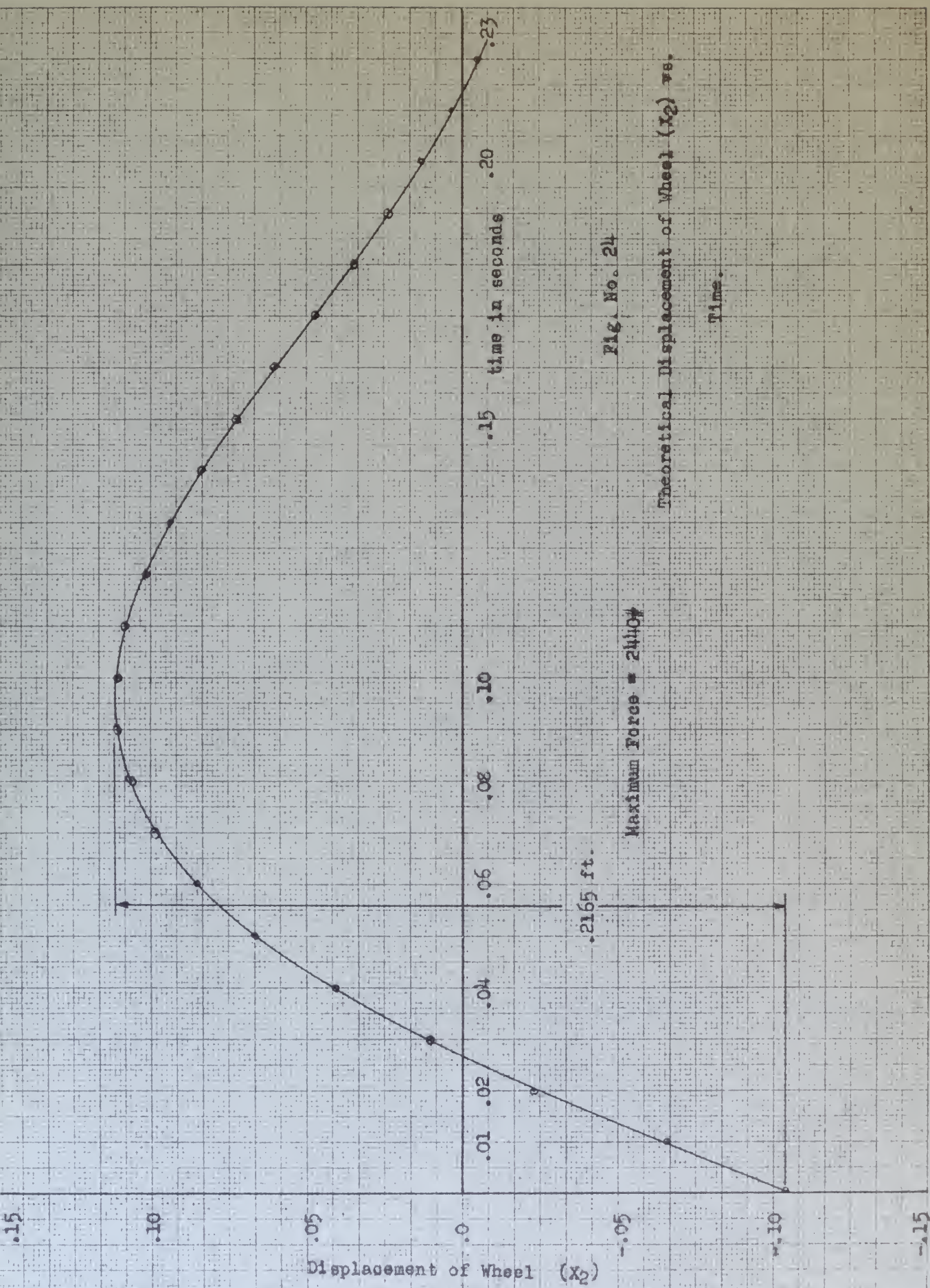
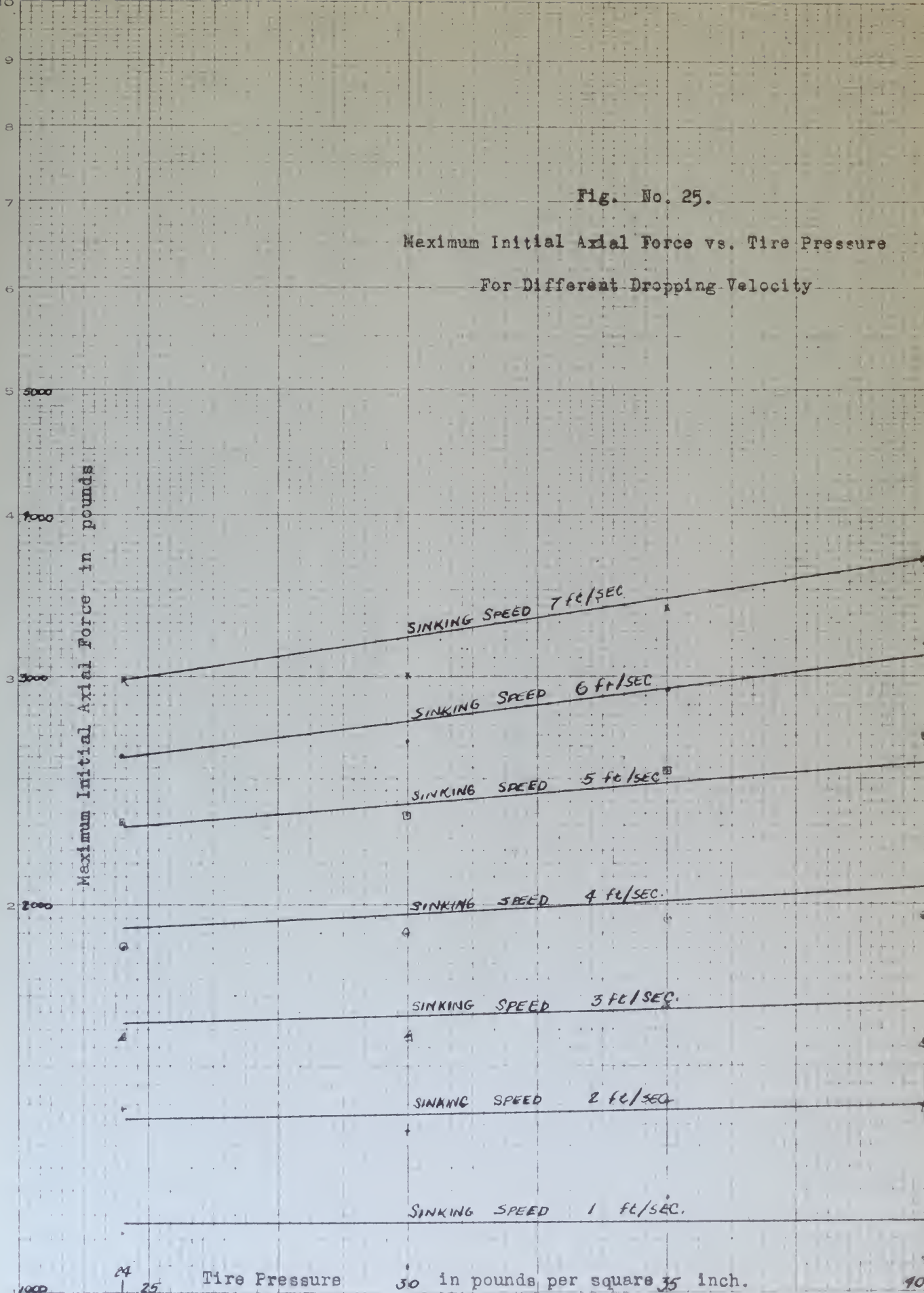






Fig. No. 25.

Maximum Initial Axial Force vs. Tire Pressure  
For Different Dropping Velocity







2800

2600

2400

2200

2000

1800

1600

1400

1300

Maximum Initial Axial Force in pounds

Fig. No. 26

Maximum Initial Axial Force

vs. Tire Pressure

For Different Dropping Velocity.

5 FPS

SINKING SPEED

SINKING

4 FPS

SINKING SPEED

SINKING

3 FPS

SINKING SPEED

SINKING

2 FPS

SINKING SPEED

SINKING

24 25

30

35

40

Tire Pressure in pounds per square inch.



## CONCLUSION

The data of this experiment indicates that the theory and the experimental results agree quite closely.

As the tire pressure increases the axial force increases. At the slower striking velocities the effect of the tire pressure is not as noticeable as when the vertical forces are of a greater magnitude. In Fig. No. 25 there is an indication that the axial force increases by the square as the dropping velocity increases linearly. On log paper (Fig. No. 25) the graph shows equal distances between the curves at constant tire pressure. These equal distances increase between the curves as the pressure of the tire becomes greater. This gives an increasing slope. Fig. No. 26 shows a better picture of this slope and indicates more clearly that as the tire pressure increases the force along the strut increases.

Both the Brush Recorder and motion picture show that a temporary frequency is introduced into the system during the first phase of landing impact of the landing gear. This frequency results from the drag forces imposed on the strut, and the vertical vibrations of the tire. The film indicates that the landing gear, after its initial contact with the flywheel, does not leave the landing surface.



# CONCLUSIONS

The data of this experiment indicates that the theory and the experimental results agree quite closely. In the first pressure measurement the axial force increased. At the same time the velocity of the air movement is not as noticeable as when the vertical forces are of a greater magnitude. In Fig. No. 32 there is an indication that the axial force increased by the same as the dropping velocity in pressure intensity. On the same (Fig. No. 32) the graph shows equal distances between the curves at constant time pressure. These equal distances increase between the curves as the pressure of the air becomes greater. This gives an increasing slope. Fig. No. 33 shows a better picture of this phenomenon and indicates more clearly that as the time pressure increases the force along the axial direction.

Both the theory and the experiment show that a frequency tendency is introduced into the system during the first phase of loading. It is of the nature of a wave. This frequency varies from the first force in phase on the start, and the vertical vibration of the first. The first indicates that the loading wave, after its initial contact with the system, does not have the loading surface.



The theoretical and experimental development points out that critical damping for the oleo had been reached.

It is recommended that a more complete study be made of the "spring constants", damping characteristics, and dynamic friction involved in the problem of landing gear.

The chemical and experimental development of the  
 out this critical thinking for the also has been needed.

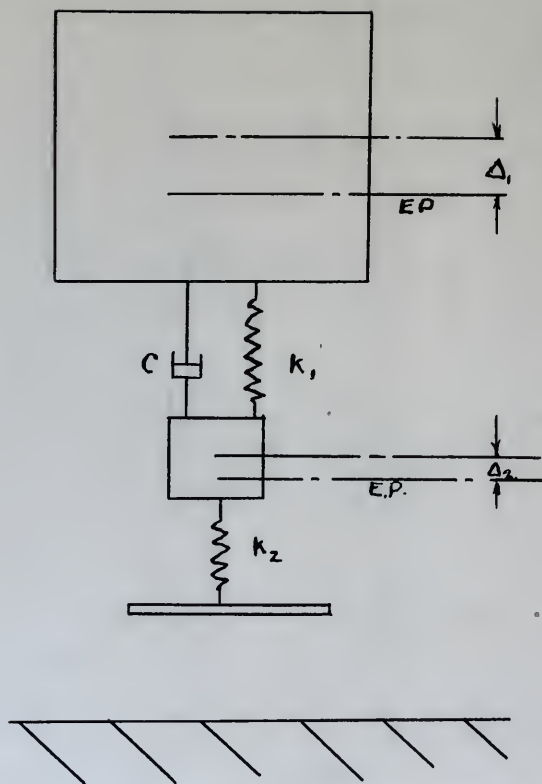
It is recommended that a more complete study be  
 made of the "Applied Chemistry," dealing with the  
 and dynamic relation involved in the process of feeding  
 food.

The following are the main points of the report:  
 1. The first point is that the study of the  
 2. The second point is that the study of the  
 3. The third point is that the study of the  
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The following are the main points of the report:

# APPENDIX



$k_1$  = spring constant of Oleo strut

$k_2$  = spring constant of tire

$c$  = damping constant of oleo strut

$W_1$  = weight of airplane

$W_2$  = weight of wheel and lower half of strut

$M$  = mass

$A, B, C, D, A', B', C', D'$  = constants

$i$  = imaginary unit

$s$  = complex frequency

From the forces acting on this diagram the equation of motion can be written.

$$(1) -m_1 \ddot{x}_1 - c(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

$$(2) -m_2 \ddot{x}_2 - k_2 x_2 + k_1(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2) = 0$$

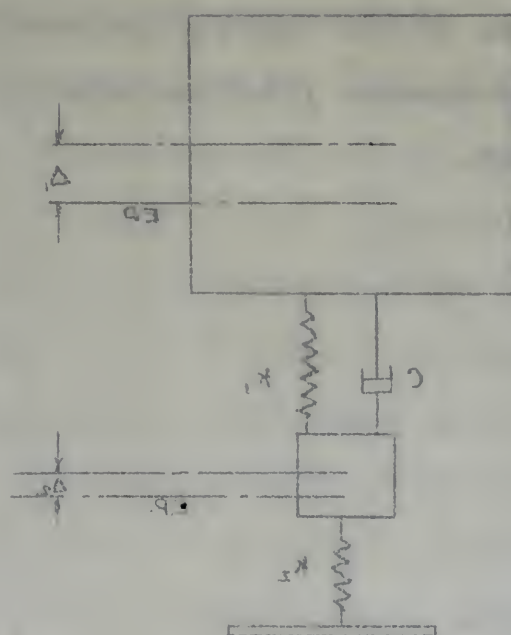
$$\begin{aligned} \text{Let: } x_1 &= A e^{st} & \dot{x}_1 &= s A e^{st} & \ddot{x}_1 &= s^2 A e^{st} \\ x_2 &= B e^{st} & \dot{x}_2 &= s B e^{st} & \ddot{x}_2 &= s^2 B e^{st} \end{aligned}$$

Subs. in (1) and (2) the following results

$$(3) (-m_1 s^2 - cs - k_1) A e^{st} + B e^{st} (cs + k_1) = 0$$

$$(4) (-m_2 s^2 - cs - k_1 - k_2) B e^{st} + A e^{st} (cs + k_1) = 0$$

# PROBLEM 10



- $h_1$  = height of upper block  
 $h_2$  = height of lower block  
 $W_1$  = weight of upper block  
 $W_2$  = weight of lower block  
 $k_1$  = spring constant of upper spring  
 $k_2$  = spring constant of lower spring  
 $c$  = dashpot coefficient  
 $\Delta_1$  = static deflection of upper spring  
 $\Delta_2$  = static deflection of lower spring  
 $\Delta$  = total static deflection

From the forces acting on each block the equation of motion can be written:

$$(1) \quad m_1 \ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k_1 x_1 + k_2 x_2 = m_1 g$$

$$(2) \quad m_2 \ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2 x_2 = m_2 g$$

$$\text{Let } x_1 = A_1 e^{i\omega t} \quad \text{and } x_2 = A_2 e^{i\omega t}$$

Substituting in (1) and (2) the following results:

$$(1) \quad (-m_1 \omega^2 + i c \omega + k_1 + k_2) A_1 - i c \omega A_2 = m_1 g$$

$$(2) \quad (-m_2 \omega^2 + i c \omega + k_2) A_2 - i c \omega A_1 = m_2 g$$



The  $cst$  cancels--Then putting into determinant form and solve for  $s$  (which is the freq.)

$$(5) \begin{vmatrix} -m_1 s^2 - cs - k_1 & cs + k_1 \\ cs + k_1 & -m_2 s^2 - cs - k_1 - k_2 \end{vmatrix} = 0$$

Solve the resultant equation

$$(7) m_1 m_2 s^4 + (m_1 + m_2)cs^3 + (k_1 m_1 + k_1 m_2 + k_2 m_1)s^2 + k_2 cs + k_1 k_2 = 0$$

$$\text{Letting } a = \frac{(m_1 + m_2)c}{m_1 m_2}$$

$$b = \frac{k_1 m_1 + k_1 m_2 + k_2 m_1}{m_1 m_2}$$

$$c = \frac{k_2 c}{m_1 m_2}$$

$$d = \frac{k_1 k_2}{m_1 m_2}$$

$$(8) s^4 + as^3 + bs^2 + cs + d = 0$$

Brots "approximate factorization" gives a relative close solution for this equation under certain conditions:

$$f(s) \approx (s^2 + as + b) \left(s^2 + \frac{c}{b}s + \frac{d}{b}\right) = 0$$

$$(9) s_{1,2} \approx \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$$(10) s_{3,4} \approx \frac{-\frac{c}{b} \pm \sqrt{\left(\frac{c}{b}\right)^2 - 4\frac{d}{b}}}{2}$$



Synthetic division give better results. With this, the following can be written:

$$(11) \quad x_1 = Ae^{s_1 t} + Be^{s_2 t} + Ce^{s_3 t} + De^{s_4 t}$$

$$(12) \quad x_2 = A'e^{s_1 t} + B'e^{s_2 t} + C'e^{s_3 t} + D'e^{s_4 t}$$

Then put A in terms of A'

$$\begin{vmatrix} (-m_1 s^2 - cs - k_1)A + (Cs + k_1)A' \\ (cs + k_1)A + (-m_2 s^2 - cs - k_1 - k_2)A' \end{vmatrix} = 0$$

or

$$(13) \quad A = \frac{cs_1 + k_1}{m_1 s_1^2 + cs_1 + k_1} A' = \phi_1 A'$$

$$(14) \quad B = \frac{cs_2 + k_1}{m_1 s_2^2 + cs_2 + k_1} B' = \phi_2 B'$$

$$(15) \quad C = \frac{cs_3 + k_1}{m_1 s_3^2 + cs_3 + k_1} C' = \phi_3 C'$$

$$(16) \quad D = \frac{cs_4 + k_1}{m_1 s_4^2 + cs_4 + k_1} D' = \phi_4 D'$$

Now set up boundary conditions

$$\begin{array}{lll} \text{at } t = 0 & x_1 = -\Delta_1 & x_2 = -\Delta_2 \\ & \dot{x}_1 = v_0 & \dot{x}_2 = v_0 \end{array}$$

where  $-\Delta_1$  and  $-\Delta_2$  equal the distance from center of mass to equilibrium position

$$-1 = \frac{(m_1 - m_2)g}{K_1} + \frac{(m_1 + m_2)g}{K_2}$$





$$-\Delta_2 = \frac{(m_1 + m_2) g}{k_2}$$

$$(17) -\Delta_1 = A e^{s_1 t} + B e^{s_2 t} + C e^{s_3 t} + D e^{s_4 t}$$

$$(18) -\Delta_2 = A' e^{s_1 t} + B' e^{s_2 t} + C' e^{s_3 t} + D' e^{s_4 t}$$

Let  $V_0$  = The dropping velocity of the landing gear  
and taking derivative, gives

$$(19) V_0 = s_1 A e^{s_1 t} + s_2 B e^{s_2 t} + s_3 C e^{s_3 t} + s_4 D e^{s_4 t}$$

$$(20) V_0 = s_1 A' e^{s_1 t} + s_2 B' e^{s_2 t} + s_3 C' e^{s_3 t} + s_4 D' e^{s_4 t}$$

Rewriting (17) and (19)

$$(21) -\Delta_1 = \phi_1 A' e^{s_1 t} + \phi_2 B' e^{s_2 t} + \phi_3 C' e^{s_3 t} + \phi_4 D' e^{s_4 t}$$

$$(22) V_0 = s_1 \phi_1 A' e^{s_1 t} + s_2 \phi_2 B' e^{s_2 t} + s_3 \phi_3 C' e^{s_3 t} + s_4 \phi_4 D' e^{s_4 t}$$

Here are four equations, four unknowns putting in determinant form:

$$\text{Let } E = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \phi_1 & \phi_2 & \phi_3 & \phi_4 \\ s_1 & s_2 & s_3 & s_4 \\ s_1 \phi_1 & s_2 \phi_2 & s_3 \phi_3 & s_4 \phi_4 \end{vmatrix}$$

$$E_1 = \begin{vmatrix} -\Delta_2 & 1 & 1 & 1 \\ -\Delta_1 & \phi_2 & \phi_3 & \phi_4 \\ V_0 & s_2 & s_3 & s_4 \\ V_0 & s_2 \phi_2 & s_3 \phi_3 & s_4 \phi_4 \end{vmatrix}; \quad E_2 = \begin{vmatrix} 1 & -\Delta_2 & 1 & 1 \\ \phi_1 & -\Delta_2 & \phi_3 & \phi_4 \\ s_1 & V_0 & s_3 & s_4 \\ s_1 \phi_1 & V_0 & s_3 \phi_3 & s_4 \phi_4 \end{vmatrix}$$

$$\frac{1}{2} (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \Delta$$

$$\frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (17)$$

$$\frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (18)$$

Let  $\Delta$  be the Laplacian, which is the Laplacian

and taking derivative, gives

$$\frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (19)$$

$$\frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (20)$$

Adding (17) and (19)

$$\frac{1}{2} + \frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (21)$$

$$+ \frac{1}{2} \frac{\partial}{\partial x} + \frac{1}{2} \frac{\partial}{\partial y} + \frac{1}{2} \frac{\partial}{\partial z} + \frac{1}{2} \frac{\partial}{\partial t} = \frac{1}{2} \Delta \quad (22)$$

Now we have two equations, from which we can find the value of  $\Delta$

which is

$$\Delta = \frac{1}{2} \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} + \frac{\partial}{\partial t} \right)^2$$

$$\Delta = \frac{1}{2} \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} + \frac{\partial}{\partial t} \right)^2 = \frac{1}{2} \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} + \frac{\partial}{\partial t} \right)^2$$

$$E_3 = \begin{vmatrix} 1 & 1 & -\Delta_2 & 1 \\ \phi_1 & \phi_2 & -\Delta_1 & \phi_4 \\ s_1 & s_2 & v_0 & s_4 \\ s_1\phi_1 & s_2\phi_2 & v_0 & s_4\phi_4 \end{vmatrix}; \quad E_4 = \begin{vmatrix} 1 & 1 & 1 & -\Delta_2 \\ \phi_1 & \phi_2 & \phi_3 & -\Delta_1 \\ s_1 & s_2 & s_3 & v_0 \\ s_1\phi_1 & s_2\phi_2 & s_3\phi_3 & v_0 \end{vmatrix}$$

Therefore:

$$A' = \frac{E_1}{E}; \quad B' = \frac{E_2}{E}; \quad C' = \frac{E_3}{E}; \quad D' = \frac{E_4}{E}$$

Substituting in (11) and (12) the equations of motion result.

$$(23) \quad x_1 = \frac{E_1}{E} \phi_1 e^{s_1 t} + \frac{E_2}{E} \phi_2 e^{s_2 t} + \frac{E_3}{E} \phi_3 e^{s_3 t} +$$

$$\frac{E_4}{E} \phi_4 e^{s_4 t}$$

$$(24) \quad x_2 = \frac{E_1}{E} e^{s_1 t} + \frac{E_2}{E} e^{s_2 t} + \frac{E_3}{E} e^{s_3 t} + \frac{E_4}{E} e^{s_4 t}$$

Let  $s_1, s_2, s_3$ , and  $s_4$  be complex roots,

Or,

$$s_1 = a_1 + i\omega_1$$

$$s_3 = a_2 + i\omega_2$$

$$s_2 = a_1 - i\omega_1$$

$$s_4 = a_2 - i\omega_2$$

Again (11) and (12) can be written

$$(25) \quad x_1 = A e^{(a_1 + i\omega_1)t} + B e^{(a_1 - i\omega_1)t} + C e^{(a_2 + i\omega_2)t} \\ + D e^{(a_2 - i\omega_2)t}$$

$$(26) \quad x_2 = A' e^{(a_1 + i\omega_1)t} + B' e^{(a_1 - i\omega_1)t} + C' e^{(a_2 + i\omega_2)t} \\ + D' e^{(a_2 - i\omega_2)t}$$

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$$\begin{vmatrix} \Delta & 1 & 1 & 1 \\ 1 & \Delta & 1 & 1 \\ 1 & 1 & \Delta & 1 \\ 1 & 1 & 1 & \Delta \end{vmatrix} = \Delta^4 - 3\Delta^2 + 2 = (\Delta-1)^2(\Delta+2)$$

$$\begin{vmatrix} \Delta & 1 & 1 & 1 \\ 1 & \Delta & 1 & 1 \\ 1 & 1 & \Delta & 1 \\ 1 & 1 & 1 & \Delta \end{vmatrix} = \Delta^4 - 3\Delta^2 + 2 = (\Delta-1)^2(\Delta+2)$$

Therefore

$$\frac{\Delta}{\Delta-1} = 1; \frac{\Delta}{\Delta+2} = 1; \frac{\Delta}{\Delta-1} = 1; \frac{\Delta}{\Delta+2} = 1$$

Substituting in (11) and (12) the values of  $\Delta$

we get

$$+ \frac{1}{2} \omega_1 \frac{\Delta}{\Delta-1} + \frac{1}{2} \omega_2 \frac{\Delta}{\Delta+2} + \frac{1}{2} \omega_3 \frac{\Delta}{\Delta-1} = 1 \quad (13)$$

$$\frac{\Delta}{\Delta-1} = 1$$

$$+ \frac{1}{2} \omega_1 \frac{\Delta}{\Delta-1} + \frac{1}{2} \omega_2 \frac{\Delta}{\Delta+2} + \frac{1}{2} \omega_3 \frac{\Delta}{\Delta-1} = 1 \quad (14)$$

Let  $\omega_1 = \omega_2 = \omega_3 = \omega$

we

$$\omega(1 + \frac{1}{2}) = 1$$

$$\omega(1 + \frac{1}{2}) = 1$$

$$\omega(1 - \frac{1}{2}) = 1$$

$$\omega(1 - \frac{1}{2}) = 1$$

Adding (11) and (12) we get

$$\frac{\Delta}{\Delta-1} + \frac{\Delta}{\Delta+2} + \frac{\Delta}{\Delta-1} + \frac{\Delta}{\Delta+2} = 2 \quad (15)$$

$$\frac{\Delta}{\Delta-1} = 1$$

$$\frac{\Delta}{\Delta-1} + \frac{\Delta}{\Delta+2} + \frac{\Delta}{\Delta-1} + \frac{\Delta}{\Delta+2} = 2 \quad (16)$$

$$\frac{\Delta}{\Delta-1} = 1$$



DeMoivre's Theorem:

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

$$e^{-i\omega t} = \cos \omega t - i \sin \omega t$$

(25) and (26) are written:

$$(27) \quad x_1 = A e^{a_1 t} e^{i\omega_1 t} + B e^{a_1 t} e^{-i\omega_1 t} + C e^{a_2 t} e^{i\omega_2 t} \\ + D e^{a_2 t} e^{-i\omega_2 t}$$

$$(28) \quad x_2 = A' e^{a_1 t} e^{i\omega_1 t} + B' e^{a_1 t} e^{-i\omega_1 t} + C' e^{a_2 t} \\ e^{i\omega_2 t} + D' e^{a_2 t} e^{-i\omega_2 t}$$

$$(29) \quad x_1 = (A \cos \omega_1 t + A i \sin \omega_1 t) e^{a_1 t} + e^{a_2 t} \\ (B \cos \omega_1 t - B i \sin \omega_1 t) + e^{a_2 t} (C \cos \omega_2 t + C i \sin \omega_2 t) + e^{a_2 t} (D \cos \omega_2 t - \\ D i \sin \omega_2 t)$$

$$(30) \quad x_2 = (A' \cos \omega_1 t + A' i \sin \omega_1 t) e^{a_1 t} + e^{a_1 t} \\ (B' \cos \omega_1 t - B' i \sin \omega_1 t) + e^{a_2 t} (C' \cos \omega_2 t + C' i \sin \omega_2 t) + e^{a_2 t} (D' \cos \omega_2 t - D' i \sin \omega_2 t)$$

Combining (29) and (30) using De Moivre's Theorem

$$(31) \quad x_1 = e^{a_1 t} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t) + e^{a_2 t} \\ (C_3 \cos \omega_2 t + C_4 \sin \omega_2 t)$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1$$

$$-i\omega_1 = \cos \omega_1 - i \sin \omega_1$$

$$(32) \text{ and (33) are written:}$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1 \quad (34)$$

$$-2i\omega_1 = \cos \omega_1 + i \sin \omega_1 - \cos \omega_1 + i \sin \omega_1 = 2i \sin \omega_1$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1 \quad (35)$$

$$-2i\omega_1 = \cos \omega_1 + i \sin \omega_1 - \cos \omega_1 + i \sin \omega_1 = 2i \sin \omega_1$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1 \quad (36)$$

$$-2i\omega_1 = \cos \omega_1 + i \sin \omega_1 - \cos \omega_1 + i \sin \omega_1 = 2i \sin \omega_1$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1$$

$$(37) \text{ and (38)}$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1 \quad (39)$$

$$-2i\omega_1 = \cos \omega_1 + i \sin \omega_1 - \cos \omega_1 + i \sin \omega_1 = 2i \sin \omega_1$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1$$

$$-2i\omega_1 = \cos \omega_1 + i \sin \omega_1 - \cos \omega_1 + i \sin \omega_1 = 2i \sin \omega_1$$

$$\text{Combining (39) and (40) using De Moivre's theorem}$$

$$2\omega_1 = \cos \omega_1 + i \sin \omega_1 + \cos \omega_1 - i \sin \omega_1 = 2 \cos \omega_1 \quad (41)$$

$$(42) \text{ and (43) are written:}$$

$$(32) \quad x_2 = e^{a_1 t} (C_1' \cos \omega_1 t + C_2' \sin \omega_1 t) + e^{a_2 t} (C_3' \cos \omega_2 t - C_4' \sin \omega_2 t)$$

where

$$C_1 = (A + B)$$

$$C_2 = (A - B)i$$

$$C_3 = (C + D)$$

$$C_4 = (C - D)i$$

$$C_1' = (A' + B')$$

$$C_2' = (A' - B')i$$

$$C_3' = (C' + D')$$

$$C_4' = (C' - D')i$$

$$+ (f_{10} \sin \theta + f_{20} \cos \theta) \sin \theta = g \quad (11)$$

$$(f_{10} \sin \theta + f_{20} \cos \theta) \sin \theta = g$$

where

$$f_1 = (1 + \mu) = 10$$

$$f_2 = (1 - \mu) = 0$$

$$f_3 = (1 + \mu) = 10$$

$$f_4 = (1 - \mu) = 0$$

$$f_5 = (1 + \mu) = 10$$

$$f_6 = (1 - \mu) = 0$$

$$f_7 = (1 + \mu) = 10$$

$$f_8 = (1 - \mu) = 0$$



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An investigation of  
dynamic stresses in a  
landing gear at a pre-  
determined strut angle.

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